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Editorial

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Nowadays, there is an increasing debate about the role and the potential associated to automation and connectivity in the transport sector. Connected and automated driving seem to be the solution for transport externalities, helping in improving safety, while reducing congestion and pollution levels.

However, if true, there would still be the need to guide the development of the transport system in order to be able to accommodate such evolution. This calls for a continuous investigation of methods, models and products by both researchers and industries, to assess the transport needs and pilot the way forward.

This collection of papers starts from these assumptions and aims to provide an overview of emerging methods, technologies and innovation trends, including both theory and practice, for shaping the future of transport systems.

A broad range of topics are covered moving from road safety and simulation to big data and data analytics for transportation modeling; urban transport is faced in terms of integration between transport systems and environment, as well as Mobility As A Service (MAAS) optimization. Finally, interesting findings are provided for both emerging rail transportation technologies and innovation in logistics and freight delivery.

Here, we briefly summarized the contents of each paper, as they appear in the issue.

The paper "Modeling merging and discretionary lane changing behaviors, a signaling game analysis" by S.R. Ramezanpour Nargesi, S. Shokoohyar and S. Mattingly aims to improve existing lane changing models adopting an enhanced game theory methodological approach. Merging and discretionary lane changing behaviors are modelled considering two players, specifically: a target vehicle which decides whether to change lane or wait for another acceptable gap and a lag vehicle which decides to accelerate (for closing the gap), decelerate (for cooperation), or to keep its current speed. Both the players are subject to safety constraints.

S.-H. Huang is the author of the paper "Examining the robustness of perception and reaction time from the perspective of driver attention allocation", where the duration of drivers' attention allocations to multiple focal points under varying conditions is investigated. This is done by adopting naturalistic driving data, finally obtaining valuable insights on road users' behavior to be adopted for both roadway design and traffic management.

In “A dynamic aggregate model for simulating network wide travel time reliability”, presented during ISETT2019, Lu and Liu propose a model that combines Two-Fluid Model, mean-standard deviation relationship of travel time rate and standardized travel time distribution in order to simulate travel time reliability. The dynamic aggregated model involves three steps, the first generates the historical travel condition by means of GPS data collected from taxi; the second computes the probability of departure time choice and, finally, the third concerns the simulation of the density evolution. The main advantages of this modeling framework deals with the ease of implementation and the need to use only taxi GPS data.

The paper “Application of Ground Penetrating Radar for mapping tree root system architecture and mass density of street trees”, presented during ISETT2019, by L. Lantini, A. M. Alani, I. Giannakis, A. Benedetto and F. Tosti deals with the assessment of the tree roots in an urban environment by means of Ground Penetrating Radar (GPR). This topic is very important considering the extensive damages that can be caused by uncontrolled development of tree roots, that can endanger safety of pedestrians, cyclists and drivers. Authors investigate the potential of GPR in detecting the tree roots and mapping the root system architecture; moreover, they propose a data processing methodology to estimate the root mass density under road pavement structures at different depths. Finally, the GPR has proven also to be useful in identifying safety-related events from the interaction between the root system and the pavement structure.

The authors of “Optimal rental and configuration of reserved parking for car sharing by Integer Linear Programming and Ant Colony Optimization”, presented during ISETT2019, represent an important case of shared mobility and consider a crucial service in modern smart cities. In this work, they highlight the relevant role that parking slots reserved to car sharing vehicles may have in favoring the success and diffusion of such services, also referring to remarkable regulations of some major cities. They propose an Integer Linear Programming model that includes whether or not to rent a cluster of parking slots to carsharing companies as central decisions, furthermore they propose a metaheuristic solution algorithm that combines an improved ant colony optimization algorithm, exploiting suitable linear relaxations of the integer model, with an exact large neighborhood search. An application test with realistic data instances referring to the city of Rome is applied.

"Operation analyses on capacity enhancement for a regional railway line in UK through the implementation of the ETCS/ERTMS Level 2 HD signalling system", presented during ISETT2019, reports about an interesting study carried out by Italferr S.p.A. (Italian Railway Group). The type of signalling system analyzed represents an innovative design solution, since it is typically adopted for High-Speed railway lines. The operation of the entire line has been conducted through a micro-simulation tool, making a specific focus on the busiest section and analyzing the terminal station capacity through a Queuing Method.

The authors of "Optimization and Simulation Approach of Containers handling operations at Intermodal Terminals", presented during ISETT2019, present a paper that deals with the problem of minimizing the reshuffling of containers in an inland intermodal terminal. They implement a solution procedure for the optimization of the reshuffles of ITUs by applying a double genetic algorithm that optimizes first the positions of the unloading ITUs and then those of the blocking ITUs that need to be reshuffled. The problem is tackled according to a simulation-optimization approach. The simulation model computes the operational costs of containers, related to storage and pick-up operations in an inland yard. The proposed optimization method has been tested on a theoretical example of realistic size.

Although we know that the volume deals with a subset of topics, we are confident that the reader should find in this selection some reference points for future developments of both transportation system theory and practice.

Many persons have contributed to making the development of this volume possible. Special thanks should be given to Dr. Lei Zhang and the scientific committee of the 2019 COTA International Symposium on Emerging Trends in Transportation (ISETT) held in Rome on October 3-5, 2019, since several papers of this issue have been selected during the symposium.

A big thanks goes to prof. Ernesto Cipriani and prof. Gaetano Fusco for trusting us and giving the opportunity to follow this issue.

Precious help was also provided by Filippo Carrese in the final editing process of the papers.

Finally, we wish to acknowledge the Editor in Chief of Advances in Transportation Studies Prof. Alessandro Calvi.

The Guest Editors

Chiara Colombaroni, Livia Mannini and Marialisa Nigro

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Modeling merging and discretionary lane changing behaviors, a signaling game analysis

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Abstract

Although many studies have been conducted in developing lane changing behavior models [3], some shortcomings still exist in this area such as considering broader traffic characteristics. The lane changing behavior contains the interactions of vehicles involved in lane changing process. The objective of this paper is to introduce a theoretical model of lane changing behavior which can capture the interactions of drivers during lane changing process. Therefore, the study conducts game theoretical approach to model merging and discretionary lane changing behaviors with two players (Target vehicle; the one wanting to change lanes and Lag vehicle; the one that will be behind the target vehicle after lane changing is completed). The current lane refers to the lane where the target vehicle begins executing a lane changing maneuver and the target lane refers to the lane where the target vehicle will finish a lane changing maneuver. This research proposes a lane changing behavior model enhancement by introducing and applying more realistic conditions to lane changing scenarios. The authors formulate the lane changing process using a Game theoretical approach and expand it to the signaling game to improve existing lane changing behavior modeling. The payoff functions of target and lag vehicles are developed by incorporating several factors including the density differences of the current lane and the target lanes. The proposed lane changing model is a theoretical lane changing model with application of game theoretical approach.

Keywords – game theory, lane-changing model, traffic flow

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1. Introduction

Recently, extensive studies have sought to model driving behaviors. Lane changing represents one of the most challenging driving behaviors to model because it depends on multiple vehicles' interactions. Lane changing behavior modeling has been studied vastly but traffic engineering scholars still attempt to improve the models. A more accurate lane changing model would improve traffic simulations and enhance the outcome of traffic operation projects. The main purpose of this paper is to improve existing lane changing models using game theoretical approach.

Since the lane changing maneuver has a noticeable role in causing congestion and collisions, accurate modelling of this behavior has a crucial role in designing traffic simulation tools. Although significant efforts have been made for developing lane changing models during recent decades, most of them do not consider some key parameters such as geometry, weather condition, and broader traffic flow [11]. Therefore, investigating and developing a more conclusive lane changing model that embraces those parameters represents a meaningful contribution.

Fig.1 - Classification of lane changing models [11]

Figure 1 [11] organizes the different lane changing models that have been investigated and developed in the past decades. Based on this figure, lane changing models are classified in microscopic, macroscopic, and hybrid models. Most of lane changing models are in microscopic level which include four main categories; Incentive Based Model, Artificial Intelligence Model, Discrete Choice Model, and Rule Based Model. The proposed model in this paper is classified as Rule Based Model. This figure demonstrates different types of lane changing models that have been developed. Rahman et al. [11] conducted a detailed review and comparisons of all developed lane changing models. They have discussed the advantages and disadvantages of each model. Here, the authors tried to show almost all types of lane changing models that have been developed. The proposed theoretical model in this study is "game theoretical" model which can consider the actions of main drivers who are involved in lane changing process and investigate their instant actions. The authors attempted to use this type of lane changing model in order to explore the interactions of drivers with each other while they change lane and model those actions. For detailed review of the lane changing models, readers are referred to [11]. In section 2.1, literature on lane changing behavior models (models other than game theory approach) are discussed and in section 2.2, the main focus of this study that is game theoretical approach in traffic behavior are discussed.

Based on these modeling approaches, lane changing behavior consists of some interactive actions. A driver decides to change lanes based on other drivers' positions and behaviors. The lane changing process does not just depend on only the target vehicle (the one attempting to change lanes), but also on the behavior of the vehicles in the target lane. The lane changing models can become more complex when they consider broader traffic conditions such as lane density [3].

Modelling rational traffic behavior requires considering the dynamic interactions between drivers and their sets of actions. Game theory seems appropriate to understand, analyze, and model the sequence of decision making [16] because it captures considering the other parties' actions and choices into one's decision making process. Some traffic behaviors contain several traffic participants' decision-making; especially the lane changing behaviors where conflicts between drivers may occur. In order to apply game theory to study lane changing behavior, the type of game (Static/Dynamic, complete/incomplete information, and cooperative/non-cooperative), number of players, set of actions for each player, and their payoff functions should be specified [7].

Overall, modelling lane changing behavior using a game theoretical approach enhances the existing lane changing models by explicit consideration of the logical actions of vehicles involved in the lane changing process. A significant modelling improvement can be implemented in microscopic traffic simulation software to achieve more realistic predictions.

The remainder of this manuscript consists of a literature review, a description of the problem being studied, the game theory model, discussion, and conclusion.

2. Literature review

In the past decades, researchers applied different approaches to model lane changing behaviors. The literature review focuses on two topics. First, the authors review lane changing models with approaches other than game theory. In the second part, the paper investigates lane changing or merging behaviors modeled by a game theoretical approach.

2.1. Lane changing behavior models (models other than game theory approach)

Gipps [4] developed a lane changing model in urban areas where traffic signals, heavy vehicles, and other obstructions may affect driving behavior. His model created a hierarchy of the lane changing process and the actions drivers need to take during the maneuver. Kesting et al. [6] developed a general model for merging and discretionary lane changing behaviors with the goal of minimizing overall braking induced by lane changes (MOBIL). In their model, the utility of a given lane and also the risk of lane changing have been considered in terms of longitudinal accelerations. This consideration helped to formulate the compact and general safety incentive criteria for symmetric and asymmetric lane changing rules. Their model only represented the last stage of lane changing, which is an operational act; however, it cannot predict the strategic or tactical steps such as vehicle acceleration or deceleration in the lane changing process. Hidas [5] also presented a model of lane changing and merging behaviors, which he names Simulation of Intelligent TRAnsport Systems (SITRAS). SITRAS considered both forced and cooperative lane changing behaviors in traffic congestion situations. Based on his research, a flow-speed relationship can be generated realistically only by forced and cooperative lane changing models. However, the SITRAS model only accounted for the immediate leader and follower vehicles and not the broader traffic characteristics such as lane density. The merging behavior was analyzed by Li Gen et al. [8] with considering eight parameters that describe the gaps, times to collision between vehicles, and the merging vehicle’s speed, which are derived from US Department of Transportation Next Generation Simulation (NGSIM) trajectory data set. Another research by Schakel at el. [12] integrated a car-following model with lane changing behavior that represented traffic better at the macroscopic level by considering traffic flow speeds of different lanes, the onset of congestion, and traffic volume of each lane. A driver’s binary decision about executing or not executing a discretionary lane changing maneuver using a Fuzzy Inference System (FIS) was developed by Balal et al. [1]. They considered four variables: “the gap between the subject vehicle and the preceding vehicle in the original lane, the gap between the subject vehicle and the preceding vehicle in the target lane, the gap between the subject vehicle and the following vehicle in the target lane, and the distance between the preceding and following vehicles in the target lanes” to answer “Is it time to begin to move into the target lane?” question.

Some studies investigated the advantages and disadvantages of different lane changing models. For instance, Moridpour et al. [10] explored the existing lane changing models in literature and investigated the strengths and weaknesses of each model. Their classification identifies two main categories of lane changing behavior models (LCBM), driving decision models and driving assistant models. Ben-Akiva [2] reviewed a series of advanced lane changing models and propose a model with more integrated drivers' behaviors. They also investigated the heterogeneity of the driver population and the correlation between driver's decisions. Rahman et al. [11] reviewed and compared lane changing models related to microscopic traffic simulations. They investigated applicable improvements of existing lane changing models.

The literature makes a few comparisons between developed models and micro simulation tools' models. For instance, Sun and Elefteriadou [13] compared their developed model and the lane changing model in CORSIM. Their study used driver behavior data to model lane changing behavior. They designed two experiments, a focus group study and an in-vehicle driving test, to collect data associated with lane changing behavior and obtain both lane changing probability and gap acceptance. They tested their model in CORSIM and compared it with the embedded lane changing model in CORSIM. They showed that their model fits the observed data better than CORSIM's under different traffic congestion levels. However, they only focused on urban arterial areas for lane changing behavior modelling. In another similar study [14] collected video recordings to differentiate lane changes between free, forced, and competitive/cooperative lane changing situations and quantified the vehicle interactions during lane changing execution.

During the lane changing maneuver, the current lane changing decision can be affected by an earlier decision making process. Choudhury et al. [3] used an on-ramp merging model in a congested freeway condition for developing a framework to model state dependency in lane changing behavior. Their proposed model used state dependency to understand the influences of previous driver decisions on the ongoing decision-making process. It also can predict the future decision-making situations. However, they just focused on lateral decisions and exclude the longitudinal behaviors of cars for modeling.

Overall, future research must investigate other criteria such as considering traffic congestion downstream in the current lane and target lane because if the driver observes any congestion downstream, then lane changing may not happen. This paper investigates adding this criterion to modeling approaches. The following section reviews, the lane changing models developed with game theory.

2.2. Game theoretical approach in traffic behavior

Recently, some research has explored lane changing modelling using game theoretical approaches. Zhang [19] presented an analysis of traffic behavior based on game theory because the traffic behaviors represent the outcome of a traffic participant's decision making process and many types of conflicts and interactions between vehicles may occur. Yao [18] also modeled the interactions of vehicles and bicyclists using a game theoretical approach. The objective of players was to keep current speed while considering safety constraints. A non-cooperative, static, strategic, and with complete information game was used to find Nash equilibrium.

Logically, game theory can model the merging process. Kita [7] modeled the behavior of merging and through cars using game theory. Both cars tried to achieve the maximum benefit by predicting the other's behaviors, which represents a two-person non-zero-sum non-cooperative game; he used video recording data to model and calibrate the lane changing process. However, he based the pay-off function for the target and lag vehicles on minimizing the risk of lane changing (according to time to collision), which neglected any speed gaining advantage for the target vehicle. Liu et al. [9] also developed a vehicle interactions model in a merging situation using a game theory approach. Their game included the freeway on-coming through vehicle and the on-ramp merging vehicle as players. These vehicles competed with each other to earn the highest revenue during the merging process. The through vehicle tried to maintain its speed and the merging vehicle tried to enter the main lane as soon as possible, which represents a non-cooperative game with adopting strategies from a Nash equilibrium.

Other than general traffic behaviors and the merging process, lane changing can be modeled by game theory. Talebpour et al. [15] proposed a lane changing model with a game theoretical approach. They modeled merging and discretionary lane changing behaviors in one framework. Their model for discretionary lane changing evaluated the lane changing benefits based on acceleration to prevent collision and also the speed gain after the maneuver. In this research, the lag vehicle also investigated whether to cooperate with the target vehicle or not. This model also investigated lane changing behavior in a connected vehicle environment. Wang et al. [17] also proposed a lane changing model that can be applied in connected and autonomous vehicle systems. They used dynamic game theory and receding horizon optimal control to develop a predictive method for lane changing and car following control. Their model evaluated the continuous accelerations and lane changing process together. Based on this study, by using human driven models and estimating the response of regular vehicles, autonomous vehicles can use information from on-board sensors and make cooperative lane changing without inter-vehicle communications.

Game theoretical approach is applicable when interactions between different players exist and decision making of each player has influence on others. Since in lane changing situations, different car drivers interact with each other and cooperation of each vehicle affect the action of other drivers, game theory technique is appropriate to be used in modeling purposes of this traffic behavior.

Although several studies have investigated lane changing behavior modeling using game theory, some shortcomings such as considering broader traffic characteristics in the payoff functions of the game players still exist. This research considers the merging case as merging lane changing (MLC) and all other lane changes as discretionary lane changing (DLC). The authors develop different payoff functions for the MLC and DLC cases in the proposed model. This paper seeks to model lane changing behavior more effectively and accurately, which the authors present in detail in the following sections.

3. Problem definition

Lane changing modeling plays a crucial role in transportation studies because this behavior plays an important role in traffic management policies and traffic safety. Traffic projects rely on using traffic simulator tools, so investigating the factors that may affect lane changing behavior, which may improve simulation results, remains critical [10].

As previously discussed, lane changing behavior combines the decision making process and many conflicts that happen between vehicles; therefore, game theory represents one of the best approaches for modeling lane changing due to the complexity of this process [19]. Previous models have failed to consider some important broader traffic characteristics such as lane density. Lane density appears to play a role in the lane changing process. For instance, if a driver observes congestion downstream in the target lane, the lane change probably does not happen even if an acceptable gap exists or speed gain may occur after lane change completion. To be clearer, in the proposed modeling approach, lane density considers the driving environment beyond the surrounding vehicles, and considers the drivers' evaluation of traffic congestion in the current lane and the target lane by monitoring conditions downstream of the drivers' current positions. Therefore, this study considers the density differences of the current lane and the target lanes as an element in the payoff functions of the target vehicle in discretionary lane changing process.

As a result, modeling lane changing behavior with game theory that investigates the effects of lag and target vehicles and also considering the broader traffic condition can improve the existing models.

4. Modeling lane changing behavior using game theory

This study models discretionary lane changing (DLC) and merging lane changing (MLC) behaviors on a freeway using a game theoretical approach. As discussed earlier, including lane density in the lane change model represents a significant improvement over existing approaches. Figures 2 to 5 represent the typical discretionary and merging lane changing process in uncongested and congested traffic situations. Figure 2 to 5 present which conditions of lane changing behaviors are to be modeled in this study. As shown in these figures, modeling lane changing maneuvers in "uncongested/discretionary", "congested/discretionary", "uncongested/mandatory" and "congested/mandatory" situations are investigated. Additionally, figures 2 to 5 represent traffic congestion (congested or uncongested) and the type of lane changing (merging or discretionary) which play an important role in understanding the developed payoff functions of the game players in section 5. For instance, figures 2 and 3 present the discretionary lane changing situations as well as the positions of the vehicles which are involved in this process (target, lead and lag vehicles) in uncongested and congested conditions respectively. Moreover, figures 4 and 5 show the merging lane changing situations as well as the positions of the vehicles which are involved in this process (target, lead and lag vehicles) in uncongested and congested conditions respectively.

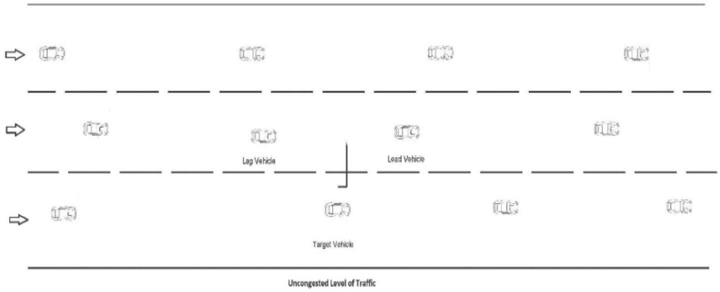

Fig. 2 - Discretionary lane changing process in uncongested traffic situation

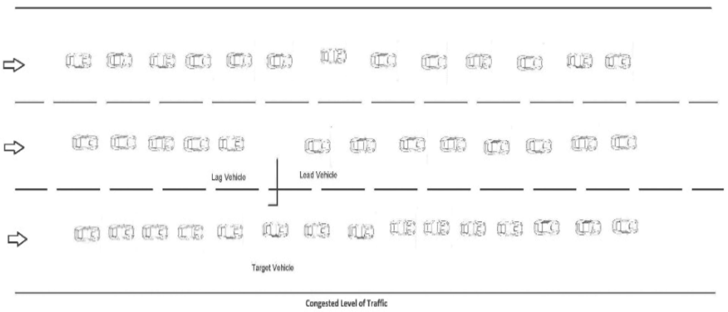

Fig. 3 - Discretionary lane changing process in congested traffic situation

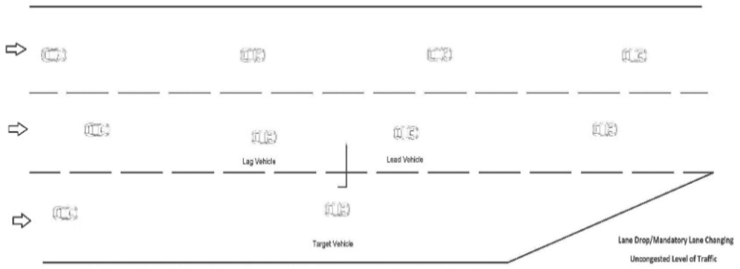

Fig. 4 - Merging lane changing process in uncongested traffic situation

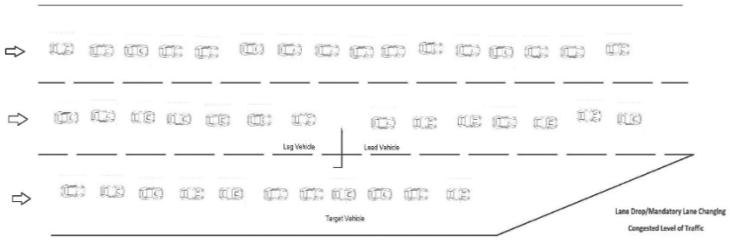

Fig. 5 - Merging lane changing process in congested traffic situation

Based on Figures 2 to 5, merging to a freeway represents a MLC, and DLC signifies all other lane changing situations. This paper expands the lane changing scenarios to include the congested and uncongested conditions for both merging and discretionary lane changes. The lane density consideration of current lane and target lane are about drivers' evaluating of traffic congestion in downstream of these lanes. These approaches to lane changing modeling is a significant contribution which can improve existing lane changing models' accuracy.

Table 1 presents the MLC game structure and Table 2 shows the game structure for DLC. Q_{ij} and R_{ij} indicate the payoffs of the target vehicle in the MLC and DLC situations, respectively. Additionally, M_{ij} and D_{ij} represent the payoffs of the lag vehicle in the MLC and DLC process, respectively. The payoffs of target and lag vehicles are discussed in detail in the following sections (i.e. Section 5, Payoff functions). For instance, Q_{11} shows the reward or the payoff of target vehicle in merging situation when the he/she changes lane and the lag vehicle accelerates. The M_{21} also represents the reward or the payoff of the lag vehicle in merging situation when he/she decelerates and target vehicle changes lane. Moreover, the R_{31} indicates the reward or payoff of target vehicle in discretionary lane changing situation when target vehicle changes lane and lag vehicle keeps its current speed.

The merging and discretionary lane changing behaviors require separate representations because the target vehicle has different payoff functions under MLC and DLC conditions, which section Payoff functions discusses in detail.

Tab. 1 - Merging lane changing behaviors game structure

Lag vehicle	Actions	Target Vehicle	
		Change lane (T_1)	Do not change lane (T_2)
	Accelerate (L1)	(Q_{11}, M_{11})	(Q_{12}, M_{12})
	Decelerate (L_2)	(Q_{21}, M_{21})	(Q_{22}, M_{22})
	Keep Current Speed (L_3)	(Q_{31}, M_{31})	(Q_{32}, M_{32})

Tab. 2 - Discretionary lane changing behaviors game structure

	Actions	Target Vehicle	
		Change lane (T_1)	Do not change lane (T_2)
Lag vehicle	Accelerate (L1)	(R_{11}, D_{11})	(R_{12}, D_{12})
	Decelerate (L_2)	(R_{21}, D_{21})	(R_{22}, D_{22})
	Keep Current Speed (L_3)	(R_{31}, D_{31})	(R_{32}, D_{32})

The authors model the traffic behaviors of the strategic players, target vehicle and lag vehicle, using the following notation. A target vehicle is aware of state of nature and (whether merging and discretionary lane changings) faces merging lane changing with a probability p and faces discretionary lane changing with a probability of $1-p$, which is common knowledge for both drivers, but only the target vehicle observes the realized state of nature. After observing the state of nature (i.e., MLC or DLC) the target vehicle decides either to change lanes denoted as T_1 or wait for another acceptable gap denoted as T_2.

Without observing the target vehicle's decision (which is inspired by [15]), the lag vehicle decides to accelerate, decelerate, or keep its current speed denoted as L_1, L_2 and L_3, respectively. Fig. 6 represents the extensive form of proposed game. This figure shows the strategic decision-makers at each of the three nodes: nature, target vehicle, and lag vehicle. The decisions are shown by the solid line and the information set is shown by the dashed line. Since the target vehicle is aware of state of nature no dashed lines are seen for the target vehicle information. The information set (dashed line) represents the fact that the lag vehicle at the time of its decision does not know the target vehicle's decision.

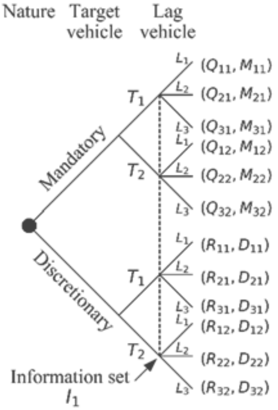

Fig. 6 - Lane changing game structure in extensive format

Tab. 3 - Lane changing game in normal format

Action	Target Vehicle			
Lag Vehicle	T_1^M	T_2^D	T_1^M	T_2^D
L_1	$(pQ_{11}+(1-p)R_{11},$ $pM_{11}+(1-p)D_{11})$	$(pQ_{11}+(1-p)R_{12},$ $pM_{11}+(1-p)D_{12})$	$(pQ_{12}+(1-p)R_{11},$ $pM_{12}+(1-p)D_{11})$	$(pQ_{12}+(1-p)R_{12},$ $pM_{12}+(1-p)D_{12})$
L_2	$(pQ_{21}+(1-p)R_{21},$ $pM_{21}+(1-p)D_{21})$	$(pQ_{21}+(1-p)R_{22},$ $pM_{21}+(1-p)D_{22})$	$(pQ_{22}+(1-p)R_{21},$ $pM_{22}+(1-p)D_{21})$	$(pQ_{22}+(1-p)R_{22},$ $pM_{22}+(1-p)D_{22})$
L_3	$(pQ_{31}+(1-p)R_{31},$ $pM_{21}+(1-p)D_{21})$	$(pQ_{31}+(1-p)R_{32},$ $pM_{31}+(1-p)D_{32})$	$(pQ_{32}+(1-p)R_{31},$ $pM_{32}+(1-p)D_{31})$	$(pQ_{32}+(1-p)R_{32},$ $pM_{32}+(1-p)D_{32})$

Based on Barron (2013), the authors convert the extensive form of the game to a normal form of the game, which is represented in Table 3. The paper denotes the targeted vehicle actions as T_a^s with subscript of target vehicle action (i.e., a, which can be change lane (1) or do not change lane (2)) and superscript of lane changing situation (i.e., s, which can be M (MLC) or D (DLC)). A tuple in each cell of Table 3 describes the expected payoff of the target and lag vehicle for a given column and row.

4.1. Extensions on the game theoretical model

In this section, the authors discuss two potential approaches for extending the game theoretical model. First, the current model tries to consider most common actions for both the target vehicle and the lag vehicle. This model may be extended by considering more actions; for instance, the lag vehicle can also choose to change lane as an action. Second, based on previous studies [15], the current model assumes that the lag vehicle does not know the target vehicle's decision. The authors relax this assumption, and model the drivers' decisions as a *signaling game*. In signaling games, one player has more information about the state of nature than the other player. The more informed player has to decide whether to signal this piece of information, and the less informed player has to decide how to respond to the signal his opponent has sent, recognizing that signals may be strategically chosen.

In the revised model, after realizing the state of nature (i.e., MLC with probability p or DLC with probability of $(1-p)$) the target vehicle selects an action from its action set of $\{T_1, T_2\}$. The target vehicle is informed about the state of nature and can convey this information to the lag vehicle by selecting a proper action. The lag vehicle observes the target vehicle's decision and then takes an action from the set of $\{L_1, L_2, L_3\}$. The structure of drivers' decisions under this extension is presented in Fig. 7. The lag vehicle becomes aware of the target vehicle's decision of T_2 at the left information set (i.e., the left dashed line) and becomes aware of T_1 at the right information set (i.e., the right dashed line).

4.2. Contribution of the proposed model

This study contributes to the existing literature on game theoretical modeling of lane changing behavior in two ways: structure of the game, and the drivers' payoff. This section mainly focuses on the structure of the games that are presented in Figure 6 and 7 and then Section 5 formulates the drivers' payoff.

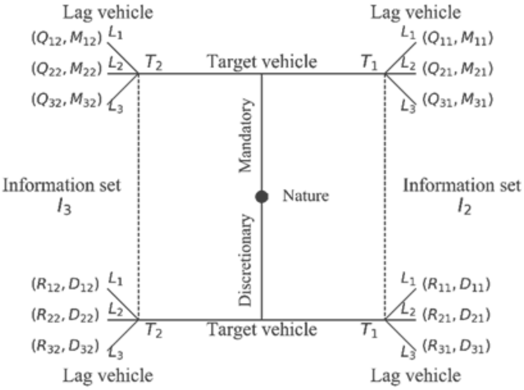

Fig. 7 - Lane changing game structure in extensive format

In both models, as presented in Figure 6 and 7, the target vehicle is aware of the state of the nature (Recall that the state of the nature is either mandatory or discretionary), however the lag vehicle is not. In the signaling game presented in Section 4.1, the target vehicle can transfer this information by taking an appropriate action as the target vehicle's action is observable by the lag vehicle in this model. The main point of contribution in the structure of the game modeled is the way and how this information (i.e. mandatory or discretionary lane changing) can be transferred from the target vehicle to the lag vehicle.

In Figure 6 (Talebpour,et al) the target vehicle action is not observed by the lag vehicle. In particular, this fact is presented by the dashed line (i.e. information set I_1 in Figure 6). That is the lag vehicle decides to accelerate (L_1), decelerate (L_2) or keep current speed (L_3) without observing the lag vehicle's action. However, in Figure 7, the lag vehicle makes decision after observing the lag vehicle action (i.e. T_1 or T_2). In particular, this fact is presented with two information sets in Figure 7 (I_1 and I_2). Being aware of the target vehicle's action at the point of decision making may reveal information in regards to the state of the nature (only known to the target vehicle) to the lag vehicle. Note that the lag vehicle belief about the state of the nature before observing the target vehicle action is that MLC is happening with probability p and DLC is occurring with probability of $(1 - p)$. After observing the target vehicle's action, the lag vehicle updates this belief following Bayesian rule. Therefore, the equilibrium concept used in signaling games is Perfect Bayesian equilibrium. There are three different categories of equilibria in the signaling games.

Pooling equilibrium: In pooling equilibrium, the target vehicle always takes action T_1 or T_2 regardless of the state of the nature. As an example assume that in the equilibrium the target vehicle always changes lane regardless of the state of the nature. In this situation, observing the target vehicle's action does not convey any information regarding the lane changing situation (i.e. MLC and DLC) to the lag vehicle. Therefore, the lag vehicle updated belief regarding the state of the nature is the same as before observing target vehicle's action.

Separating equilibrium: In Separating equilibrium, the target vehicle takes a different action at different situations (MLC or DLC). For instance, assume that the target vehicle changes lane in the mandatory lane changing situation and does not in the discretionary lane changing situation. Based on this strategy by the target vehicle, therefore the lag vehicle can conclude that the state of the nature is MLC if T_1 is observed and is DLC if T_2 is observed and can update its belief accordingly. Therefore, in this equilibrium the target vehicle can truthfully convey the state of the nature to the lag vehicle by its action.

Semi-separating equilibrium: This equilibrium is also knows as partial-pooling. This equilibrium basically is a hybrid between pooling (target vehicle's action does not reveal any information) and separating equilibrium (target vehicle's action reveals state of the nature). In this equilibrium, target vehicle at a given state of the nature changes lane following a specific probability and does not otherwise. In this equilibrium the lag vehicle can update its belief about the state of the nature based on this specific probability.

In the signaling game notion, the action taken by the target vehicle is call a *message* or a *signal* that is sent by the target vehicle to the lag vehicle as it can transfer information regarding the state of the nature to the lag vehicle. Note that what equilibrium may exist depend on the payoffs and in the most signaling games there exist multiple equilibria. Note that transferring this information is not possible in model presented in Figure 6 as the target vehicle's action is not observable to the lag vehicle in this model.

The extended game theoretical model in Section 4.1 proposes an advanced and improved lane changing model using signaling game theoretical approach. This approach makes the lane changing behavior modeling much closer to the reality in comparison with other existing models. The reason is because the target vehicle knows if he/she is in a mandatory or discretionary lane changing situation and by taking an appropriate action can signal the state of the nature (MLC or DLC) to the lag vehicle.

5. Payoff functions

This study formulates payoff functions based on the different interests of each player and considering MLC and DLC situations while both players consider safety constraints. Additionally, the target vehicle tries to minimize the time spent in the current lane in the MLC situation as well as gain speed after a discretionary lane changing maneuver. As indicated in previous sections, lane density matters in the discretionary lane changing process. Even if the target vehicle can increase its speed in a short period of time, but congestion occurs downstream, then the target vehicle may not execute a lane change. Therefore, the target vehicle must evaluate the lane density difference between the current lane and adjacent lanes. The lag vehicle seeks to minimize speed variation subject to safety constraints in both the MLC and DLC situations.

5.1. Payoff function of target vehicle in discretionary lane changing situation

In the DLC process, the target vehicle has two actions. When this player attempts to change lanes and the lag vehicle accelerates or decelerates, the player evaluates the acceleration of target vehicle for executing the lane change and the acceleration of the lag vehicle for avoiding a collision. The target vehicle also checks the difference of speed and lane density between its current lane and the target lane. When the lag vehicle keeps its current speed, the target vehicle checks all variables mentioned above except the acceleration of lag vehicle, which is 0. The other action of the target vehicle is to not change lanes. In this situation and when the lag vehicle is accelerating or decelerating, the target vehicle evaluates the acceleration of the lag vehicle for avoiding collision, the difference of the speed and lane density between its current lane and the target lane, and the waiting time that the target vehicle spends in the current lane to find another acceptable gap. However, when the lag vehicle keeps its current speed, the target vehicle checks all the previous variables except the acceleration of lag vehicle, which is 0. Below is the payoff function formulation for target vehicle:

$$R_{11}= \alpha_1+ \alpha_2A_t+ \alpha_3A_l+ \alpha_4\Delta V+ \alpha_5\Delta K+ \mu_1 \quad (1)$$
$$R_{21}= \alpha_6+ \alpha_7A_t+ \alpha_8A_l+ \alpha_9\Delta V+ \alpha_{10}\Delta K+ \mu_2 \quad (2)$$
$$R_{31}= \alpha_{11}+ \alpha_{12}A_t+ \alpha_{13}\Delta V+ \alpha_{14}\Delta K+ \mu_3 \quad (3)$$
$$R_{12}= \alpha_{15}+ \alpha_{16}A_l+ \alpha_{17}\Delta K+ \alpha_{18}\Delta V+ \alpha_{19}t_w+ \mu_4 \quad (4)$$
$$R_{22}= \alpha_{20} + \alpha_{21}A_l +\alpha_{22}\Delta K+ \alpha_{23}\Delta V+ \alpha_{24}t_w+ \mu_5 \quad (5)$$
$$R_{32}= \alpha_{25} + \alpha_{26}\Delta K+ \alpha_{27}\Delta V+ \alpha_{28}t_w+ \mu_6 \quad (6)$$

where:

R_{ij}= Payoff of target vehicle in DLC situation
A_t= Acceleration of target vehicle during lane changing process (ft/s^2)
A_l= Acceleration of lag vehicle to avoid collision (ft/s^2)
ΔV= Speed difference of current lane (initial lane of target vehicle) and target lane in (mi/hr)
ΔK= Lane density difference of current lane and target lane (veh/mi)
t_w= Waiting time of target vehicle to find another acceptable gap (s)
α_i= Parameters to be predicted by model calibration
μ_i= Term for finding unobserved variables

Table 4 shows the matrix of payoff functions of the target vehicle in DLC situation.

Tab. 4 - Target vehicle payoff functions in DLC process

Lag vehicle	Target Vehicle		
	Actions	Change lane (T_1)	Do not change lane (T_2)
	Accelerate (L_1)	$\alpha_1 + \alpha_2 A_t + \alpha_3 A_l + \alpha_4 \Delta V + \alpha_5 \Delta K + \mu_1$	$\alpha_{15} + \alpha_{16} A_l + \alpha_{17} \Delta K + \alpha_{18} \Delta V + \alpha_{19} t_w + \mu_4$
	Decelerate (L_2)	$\alpha_6 + \alpha_7 A_t + \alpha_8 A_l + \alpha_9 \Delta V + \alpha_{10} \Delta K + \mu_2$	$\alpha_{20} + \alpha_{21} A_l + \alpha_{22} \Delta K + \alpha_{23} \Delta V + \alpha_{24} t_w + \mu_5$
	Keep Current Speed(L_3)	$\alpha_{11} + \alpha_{12} A_t + \alpha_{13} \Delta V + \alpha_{14} \Delta K + \mu_3$	$\alpha_{25} + \alpha_{26} \Delta K + \alpha_{27} \Delta V + \alpha_{28} t_w + \mu_6$

Tab. 5 - Target vehicle payoff functions in MLC process

Lag vehicle	Target Vehicle		
	Actions	Change lane (T_1)	Do not change lane (T_2)
	Accelerate (L_1)	$\alpha_{29} + \alpha_{30} A_t + \alpha_{31} A_l + \mu_7$	$\alpha_{37} + \alpha_{38} A_l + \alpha_{39} t_w + \mu_{10}$
	Decelerate (L_2)	$\alpha_{32} + \alpha_{33} A_t + \alpha_{34} A_l + \mu_8$	$\alpha_{40} + \alpha_{41} A_l + \alpha_{42} t_w + \mu_{11}$
	Keep Current Speed(L_3)	$\alpha_{35} + \alpha_{36} A_t + \mu_9$	$\alpha_{43} + \alpha_{44} t_w + \mu_{12}$

5.2. Payoff function of target vehicle in merging lane changing situation

In the MLC situation, when the target vehicle changes lanes and the lag vehicle accelerates or decelerates, the target vehicle observes its own acceleration as well as the lag vehicle's. However, when the lag vehicle keeps its current speed, the target vehicle just evaluates its own acceleration. In the case of no lane change, when the lag vehicle accelerates or decelerates, the target vehicle observes the lag vehicle's acceleration as well as the waiting time in the current lane for another acceptable gap. However, when the lag vehicle keeps its current speed, the target vehicle just evaluates the waiting time in the current lane for another acceptable gap. The Q_{ij} is the payoff of target vehicle in the MLC situation. All other variables remain the same. Table 5 demonstrates the payoff functions for the target vehicle in MLC situation.

$$Q_{11} = \alpha_{29} + \alpha_{30} A_t + \alpha_{31} A_l + \mu_7 \tag{8}$$

$$Q_{21} = \alpha_{32} + \alpha_{33} A_t + \alpha_{34} A_l + \mu_8 \tag{9}$$

$$Q_{31} = \alpha_{35} + \alpha_{36} A_t + \mu_9 \tag{10}$$

$$Q_{12} = \alpha_{37} + \alpha_{38} A_l + \alpha_{39} t_w + \mu_{10} \tag{11}$$

$$Q_{22} = \alpha_{40} + \alpha_{41} A_l + \alpha_{42} t_w + \mu_{11} \tag{12}$$

$$Q_{32} = \alpha_{43} + \alpha_{44} t_w + \mu_{12} \tag{13}$$

5.3. Payoff function of lag vehicle

The structure of payoff functions of the lag vehicle does not differ in the MLC or DLC situations. However, the alphas may differ in merging and discretionary lane changing situations and thus the payoffs will be different based on nature of lane changing. It is also possible to separate the payoff functions of lag vehicles in merging and discretionary lane changing situations similar to the target vehicle, but since their structures are the same, it is omitted for space. During the lane changing process, the lag vehicle has three actions. When the target vehicle is changing lane and the lag vehicle accelerates or decelerates, then the lag vehicle evaluates its own acceleration as well as the target vehicle's acceleration for preventing a collision. However, when the lag vehicle keeps its current speed, the lag vehicle only evaluates the acceleration of the target vehicle. In the case where the target vehicle does not change lane and lag vehicle accelerates or decelerates, the lag vehicle evaluates its own acceleration, but when the lag vehicle keeps its current speed, a constant parameter and unobserved variables form the payoff function. M_{ij} and D_{ij} represent the lag vehicle payoffs in MLC and DLC situations, respectively while all other variables remain the same.

Tab. 6 - Lag vehicle payoff functions

		Target Vehicle	
Lag vehicle	Actions	Change lane (T_1)	Do not change lane (T_2)
	Accelerate (L_1)	$\alpha_{45}+\alpha_{46}A_t+\alpha_{47}A_l+\mu_{13}$	$\alpha_{53}+\alpha_{54}A_l+\mu_{16}$
	Decelerate (L_2)	$\alpha_{48}+\alpha_{49}A_t+\alpha_{50}A_l+\mu_{14}$	$\alpha_{55}+\alpha_{56}A_l+\mu_{17}$
	Keep Current Speed(L_3)	$\alpha_{51}+\alpha_{52}A_t+\mu_{15}$	$\alpha_{57}+\mu_{18}$

M_{11} or $D_{11}=\alpha_{45}+\alpha_{46}A_t+\alpha_{47}A_l+\mu_{13}$ (14)
M_{21} or $D_{21}=\alpha_{48}+\alpha_{49}A_t+\alpha_{50}A_l+\mu_{14}$ (15)
M_{31} or $D_{31}=\alpha_{51}+\alpha_{52}A_t+\mu_{15}$ (16)
M_{12} or $D_{12}=\alpha_{53}+\alpha_{54}A_l+\mu_{16}$ (17)
M_{22} or $D_{22}=\alpha_{55}+\alpha_{56}A_l+\mu_{17}$ (18)
M_{32} or $D_{32}=\alpha_{57}+\mu_{18}$ (19)

Table 6 shows the matrix of payoff functions for the lag vehicle.

6. Conclusion

This study proposes a model of merging and discretionary lane changing behavior in one framework. The authors introduce a more logical and realistic methodological approach for modeling lane changing behavior where the target vehicle is aware of the state of nature. The target vehicle decides whether to change lane or wait for another acceptable gap. Then, the lag vehicle also decides to accelerate (for closing the gap), decelerate (for cooperation), or to keep its current speed. In this game, the lag vehicle tries to minimize speed variation subject to safety constraints, while the target vehicle aims to minimize the time spent in its current lane as well as gaining speed under safety constraints. The authors propose the payoff functions based on these goals, for the target and lag vehicles.

This research attempts to improve existing lane changing models and create a more realistic representation of the lane changing process by considering merging scenario for MLC, the traffic congestion of current and target lanes, and also different payoff functions for the MLC and DLC situations.

The main aim of this paper is to introduce an enhanced game theory methodological approach for modeling merging and discretionary lane changing behaviors. This developed game theory problem does not have a unique optimal set of actions for target and lag vehicles. This work can be further extended by utilizing trajectory data sets such as NGSIM data or even a lab experimental design to find the optimal solution for the game problem which is left for future research. The other potential future research is to test this model empirically as well as applying the model into simulation environments to compare with other embedded lane changing models in traffic simulation software. The comparison of the calibrated lane changing model with other existing models can be conducted using trajectory data or other types of naturalistic driving type data sets.

Additionally, some shortcomings still exist such as considering the lead vehicle as a player or assuming more actions for game players, especially lag vehicle which is left for future work. Future enhancements to this model (one shot game) may consider continuous game theory application for modeling lane changing behavior and define multiple games for this behavior.

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