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The True Probability of a Confidence Interval

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ISBN 978-88-255-2022-4

I edition: December 2018

Contents

Introduction

The calculation of a confidence interval is, together with the hypothesis testing, the more known procedure of inferential statistics. This procedure determines, by means of a sample, a confidence (i.e. probability) that a parameter of the inherent population is contained in an arbitrary part (e.g. interval) of the real line.

However the perplexity of this probability is immediately revealed by the fact that the replacement of the said sample with an its subset determines a probability generally different and equally credible of the same event. Moreover many authors believe that the confidence in question is not a real probability (but, in truth, with arguments that do not seem decisive in front of the logical coherence of the following deductions).

In this work a remedy to this situation is achieved by defining a probability that, among all those of the same event, stands out as true and hence not is merely conventional, and that in the case of the confidence interval, also if not is exactly calculable, is however unlimitedly approximable.

For this purpose, it is preliminarily dedicated much care in defining the symbology and the concepts of logic and set theory needed for the subsequent deductions, substantially taking again notions of [1] such as the original algorithmic definitions of relations and operations between sets, the unusual formulation concerning the equality between the intersection of products and the product of intersections, and an expanded form of the important tautology that includes the *law of contraposition*.

The treatment of events and probabilities exposed in [1] is summarized, simplified and integrated by new decisive positions. In particular two important probabilities are deduced from properties of the composite events. It is thoroughly analyzed the event constituted by the happen an unknown constant into a certain part of the real line and its probability, because fundamental for the treatment of the confidence interval which is then deduced and specified in detail for the two cases, of great importance in the experimental sciences, when the statistical parameter is the mean or the variance of a normal random variable.

Chapter I

Preliminaries of Logic and Set Theory

In relation to the following logical concepts, reference is made to [1, 2, 3, 4].

A proposition is a sequence of graphic symbols. A name is a proposition that relates and represents a certain object, which alone expresses a meaning (e.g. "home") or not (e.g. "A"), and which attributes to such object the properties indicated by its eventual meaning. An object is identified by the set of all its properties.

An $\mathrm{A} \equiv \mathrm{B}$ affirms that A and B are two names of a same object and thereby reciprocally replaceable. Consequently an $\mathrm{A} \equiv \mathrm{B}$ implies that A has also the possible meaning of B (and *vice versa*).

A pairing of two names A and B is a third name (e.g. A_B) that has both meanings of the other two, therefore if A has a meaning then this is also of A_B (and analogously for B).

In identifying the members of an expression, each "$\equiv$" is considered, coherently with the parentheses, at last (and analogously "$\not\equiv$", "$=$", "$\neq$"). Is intended $\mathrm{A}\langle\mathrm{B}\rangle \equiv \mathrm{A}_\mathrm{B}$, $\wedge \equiv$ AND $\equiv$ "conjunction", $\vee \equiv$ OR $\equiv$ "inclusive disjunction", $\veebar \equiv$ XOR $\equiv$ "exclusive disjunction".

The parentheses "$\{\}$" or "$\lbrace\!\rbrace$" are used to delimit respectively a proposition generic or that defines an event. Being $\mathcal{P}$, $\mathcal{P}_\mathrm{A}$ and $\mathcal{P}_\mathrm{B}$ three propositions, is meant

$$\{\mathcal{P}_\mathrm{A} \parallel \mathcal{P}_\mathrm{B}\} \equiv \text{“}\mathcal{P}_\mathrm{A} \text{ subjected to the condition } \mathcal{P}_\mathrm{B}\text{”} \equiv \text{“}\mathcal{P}_\mathrm{A} \text{ of which } \mathcal{P}_\mathrm{B}\text{”} \equiv \text{“}\mathcal{P}_\mathrm{A} \text{ where } \mathcal{P}_\mathrm{B}\text{”}$$

$$\left\{\mathcal{P}\right\} \equiv \text{“}\mathcal{P}\text{ is true”} \quad \neg\left\{\mathcal{P}\right\} \equiv \text{“}\mathcal{P}\text{ is false”}$$

$$\neg\mathcal{P} \equiv \text{“the proposition true if } \neg\left\{\mathcal{P}\right\} \text{ and false if } \left\{\mathcal{P}\right\}\text{”}$$

$$\{\mathcal{P}_\mathrm{A} \equiv \mathcal{P}_\mathrm{B}\} \equiv \{\neg\mathcal{P}_\mathrm{A} \equiv \neg\mathcal{P}_\mathrm{B}\}$$

$$\text{“Is implicit } \mathcal{P}_\mathrm{B}\text{”} \equiv \left\{\mathcal{P}_\mathrm{A} \equiv \{\mathcal{P}_\mathrm{A} \parallel \mathcal{P}_\mathrm{B}\}\,;\forall\mathcal{P}_\mathrm{A}\right\} \tag{1.1}$$

and $\mathscr{P}\langle \mathcal{P}_\mathrm{B} \mid \mathcal{P}_\mathrm{A}\rangle$ a set of propositions from which is logically deducible $\mathcal{P}_\mathrm{B}$ being such propositions all true except $\mathcal{P}_\mathrm{A}$ that can be true or false.

Indicating $\rightarrow$ and $\Rightarrow$ the two logical connectives called respectively *entailment* or *logical implication* or *logical consequence* and *material conditional* or *material implication* or *material consequence*, is placed

$$\begin{aligned}\{\mathcal{P}_\mathrm{A} \rightarrow \mathcal{P}_\mathrm{B}\} \equiv \{\mathcal{P}_\mathrm{B} \leftarrow \mathcal{P}_\mathrm{A}\} \equiv \exists\mathscr{P}\langle \mathcal{P}_\mathrm{B} \mid \mathcal{P}_\mathrm{A}\rangle &\equiv \\ \text{“}\mathcal{P}_\mathrm{B}\text{ is logically deducible from }\mathcal{P}_\mathrm{A}\text{”} &\equiv \\ \text{“an argumentation leads from }\mathcal{P}_\mathrm{A}\text{ to }\mathcal{P}_\mathrm{B}\text{”} &\equiv \\ \text{“}\mathcal{P}_\mathrm{B}\text{ is logically demonstrable starting from }\mathcal{P}_\mathrm{A}\text{”} &\equiv \\ \text{“}\mathcal{P}_\mathrm{B}\text{ is a logical consequence of }\mathcal{P}_\mathrm{A}\text{”}&\end{aligned}$$

$$\{\mathcal{P}_\mathrm{A} \leftrightarrow \mathcal{P}_\mathrm{B}\} \equiv \{\mathcal{P}_\mathrm{A} \rightarrow \mathcal{P}_\mathrm{B}\} \wedge \{\mathcal{P}_\mathrm{A} \leftarrow \mathcal{P}_\mathrm{B}\}$$

$$\{\mathcal{P}_\mathrm{A} \Rightarrow \mathcal{P}_\mathrm{B}\} \equiv \{\mathcal{P}_\mathrm{B} \Leftarrow \mathcal{P}_\mathrm{A}\} \equiv \left\{\left\{\mathcal{P}_\mathrm{A}\right\} \rightarrow \left\{\mathcal{P}_\mathrm{B}\right\}\right\} \Leftarrow \{\mathcal{P}_\mathrm{A} \rightarrow \mathcal{P}_\mathrm{B}\}$$

$$\begin{aligned}\{\mathcal{P}_\mathrm{A} \Leftrightarrow \mathcal{P}_\mathrm{B}\} \equiv \{\mathcal{P}_\mathrm{A} \Rightarrow \mathcal{P}_\mathrm{B}\} \wedge \{\mathcal{P}_\mathrm{A} \Leftarrow \mathcal{P}_\mathrm{B}\} &\equiv \\ \left\{\left\{\mathcal{P}_\mathrm{A}\right\} \leftrightarrow \left\{\mathcal{P}_\mathrm{B}\right\}\right\} \equiv \{\mathcal{P}_\mathrm{A} \equiv \mathcal{P}_\mathrm{B}\}\ .&\end{aligned} \tag{1.2}$$

A $\mathcal{P}_\mathrm{A} \Rightarrow \mathcal{P}_\mathrm{B}$ is a $\mathscr{P}\langle \mathcal{P}_\mathrm{B} \mid \mathcal{P}_\mathrm{A}\rangle$ of which is considered conventionally only $\mathcal{P}_\mathrm{A}$, in the sense that all its propositions certainly true (i.e. all except $\mathcal{P}_\mathrm{A}$) are implicitly treated as such and are then contextually ignored as obvious. This highlights immediately $\{\mathcal{P}_\mathrm{A} \Rightarrow \mathcal{P}_\mathrm{B}\} \Rightarrow \exists\mathscr{P}\langle \mathcal{P}_\mathrm{B} \mid \mathcal{P}_\mathrm{A}\rangle$. Moreover this identity of $\mathcal{P}_\mathrm{A} \Rightarrow \mathcal{P}_\mathrm{B}$ and the always possible faculty of considering as said conventionally the only $\mathcal{P}_\mathrm{A}$ of a $\mathscr{P}\langle \mathcal{P}_\mathrm{B} \mid \mathcal{P}_\mathrm{A}\rangle$ show also $\exists\mathscr{P}\langle \mathcal{P}_\mathrm{B} \mid \mathcal{P}_\mathrm{A}\rangle \Rightarrow \{\mathcal{P}_\mathrm{A} \Rightarrow \mathcal{P}_\mathrm{B}\}$. Therefore is had

$\mathcal{P}_A \Rightarrow \mathcal{P}_B \equiv \exists\mathcal{P}\langle \mathcal{P}_B | \mathcal{P}_A \rangle$. This and the said $\mathcal{P}_A \rightarrow \mathcal{P}_B \equiv \exists\mathcal{P}\langle \mathcal{P}_B | \mathcal{P}_A \rangle$ entail $\mathcal{P}_A \Rightarrow \mathcal{P}_B \equiv \mathcal{P}_A \rightarrow \mathcal{P}_B$.

In conformity with the (2.1.1.1) of [1] is had

$$\begin{aligned}\{\mathcal{P}_A \Rightarrow \mathcal{P}_B\} \equiv \{\neg\mathcal{P}_B \Rightarrow \neg\mathcal{P}_A\} \equiv \text{“}\{\mathcal{P}_A\} \text{ is sufficient for } \{\mathcal{P}_B\}\text{”} \equiv \\ \text{“}\{\mathcal{P}_B\} \text{ is necessary for } \{\mathcal{P}_A\}\text{”} \equiv \text{“}\{\mathcal{P}_B\} \text{ if } \{\mathcal{P}_A\}\text{”} \equiv \\ \text{“}\{\mathcal{P}_A\} \text{ only if } \{\mathcal{P}_B\}\text{”} \equiv \{\mathcal{P}_A \equiv \{\mathcal{P}_A \parallel \mathcal{P}_B\}\} \equiv \{\mathcal{P}_B; \forall\mathcal{P}_A\} \equiv \exists\mathcal{P}\langle \mathcal{P}_B | \mathcal{P}_A \rangle \equiv \\ \text{“from } \mathcal{P}_A \text{ follows } \mathcal{P}_B\text{”} \equiv \text{“}\mathcal{P}_A \text{ entails } \mathcal{P}_B\text{”} \equiv \text{“}\mathcal{P}_A \text{ show } \mathcal{P}_B\text{”} \equiv \\ \text{“}\mathcal{P}_A \text{ gives rise to } \mathcal{P}_B\text{”} \equiv \text{“}\mathcal{P}_A \text{ highlights } \mathcal{P}_B\text{”} \equiv \text{“}\mathcal{P}_A \text{ implies } \mathcal{P}_B\text{”} \equiv \\ \text{“}\mathcal{P}_B \text{ is due to } \mathcal{P}_A\text{”} \equiv \text{“}\mathcal{P}_B \text{ is obtainable from } \mathcal{P}_A\text{”} \equiv \\ \text{“}\mathcal{P}_B \text{ is a direct consequence of } \mathcal{P}_A\text{”}\end{aligned} \tag{1.3}$$

whose

$$\begin{aligned}&\{\mathcal{P}_A \Rightarrow \mathcal{P}_B\} \equiv \\ &\text{“}\{\mathcal{P}_A\} \text{ is sufficient for } \{\mathcal{P}_B\}\text{”} \equiv \text{“}\{\mathcal{P}_B\} \text{ is necessary for } \{\mathcal{P}_A\}\text{”}\end{aligned}$$

is in [5], whose parentheses “{ }” can evidently be removed without risk of misunderstandings, and that, on the basis of $\mathcal{P}_A \Rightarrow \mathcal{P}_B \equiv \mathcal{P}_A \rightarrow \mathcal{P}_B$, includes the tautology $\mathcal{P}_A \rightarrow \mathcal{P}_B \equiv \neg\mathcal{P}_B \rightarrow \neg\mathcal{P}_A$ known as *law of contraposition* (a tautology is a proposition always true anyway are changed its variable arguments).

The (1.3) and (1.2) give rise to

$$\begin{aligned}\text{“}\mathcal{P}_A \text{ is necessary and sufficient for } \mathcal{P}_B\text{”} \equiv \\ \text{“}\mathcal{P}_A \text{ if and only if } \mathcal{P}_B\text{”} \equiv \text{“}\mathcal{P}_A \text{ is equivalent to } \mathcal{P}_B\text{”} \equiv \\ \text{“}\mathcal{P}_A \text{ means } \mathcal{P}_B\text{”} \equiv \{\mathcal{P}_A \equiv \mathcal{P}_B\}\end{aligned} \tag{1.4}$$

whose subscripts are exchangeable in each of the four members.

The (1.3) entails that $\mathcal{P}_A \Rightarrow \mathcal{P}_B$ and $\neg\{\mathcal{P}_B\}$ give rise to $\neg\{\mathcal{P}_A\}$, and hence entails also the kind of argumentation known as *demonstratio per absurdum* and consisting in the deduce $\{\mathcal{P}_A\}$ from $\neg\mathcal{P}_A \Rightarrow \mathcal{P}_B$ and $\neg\{\mathcal{P}_B\}$ or $\neg\{\mathcal{P}_A\}$ from $\mathcal{P}_A \Rightarrow \mathcal{P}_B$ and $\neg\{\mathcal{P}_B\}$ (and consistent thus ultimately in the establish false a $\mathcal{P}_A$ which implies a $\mathcal{P}_B$ false).

Is placed

$$\Big\{\text{from: } \mathrm{A}_1; \mathrm{A}_2; \ldots; \mathrm{A}_{\hat{\imath}};$$
$$\text{follows } \mathrm{B}_0 \diamond_1 \mathrm{B}_1 \diamond_2 \mathrm{B}_2 \cdots \diamond_{\hat{\imath}} \mathrm{B}_{\hat{\imath}} \diamond_{\hat{\imath}+1} \mathrm{B}_{\hat{\imath}+1} \cdots \diamond_{\hat{\imath}+\hat{\jmath}} \mathrm{B}_{\hat{\imath}+\hat{\jmath}}\Big\} \equiv$$
$$\Big\{\mathrm{A}_1 \Rightarrow \{\mathrm{B}_0 \diamond_1 \mathrm{B}_1\}; \mathrm{A}_2 \Rightarrow \{\mathrm{B}_1 \diamond_2 \mathrm{B}_2\}; \ldots; \mathrm{A}_{\hat{\imath}} \Rightarrow \{\mathrm{B}_{\hat{\imath}-1} \diamond_{\hat{\imath}} \mathrm{B}_{\hat{\imath}}\}\Big\}$$

where: each of $\{\diamond_1, \diamond_2, \ldots, \diamond_{\hat{\imath}+\hat{\jmath}}\}$ is a generally different relational symbol, as for example one of $\{\equiv, \not\equiv, \neq, \Rightarrow\}$; $\{\diamond_{\hat{\imath}+1}\mathrm{B}_{\hat{\imath}+1} \cdots \diamond_{\hat{\imath}+\hat{\jmath}} \mathrm{B}_{\hat{\imath}+\hat{\jmath}}\}$ may be absent and if is present the validity of its presence is considered evident; each of $\{\mathrm{A}_1, \mathrm{A}_2, \ldots, \mathrm{A}_{\hat{\imath}}\}$ is replaced by symbol "þ" when is considered evident the validity of the corresponding element of $\{\{\mathrm{B}_0 \diamond_1 \mathrm{B}_1\}, \{\mathrm{B}_1 \diamond_2 \mathrm{B}_2\}, \ldots, \{\mathrm{B}_{\hat{\imath}-1} \diamond_{\hat{\imath}} \mathrm{B}_{\hat{\imath}}\}\}$.

Is implicit

$$Æ\langle \mathrm{A} \,/\!\!/\, \mathrm{B} \,/\!\!/\, \mathrm{C}\rangle \equiv \text{“the being A a specification of B of which C”}$$

where "$/\!\!/\,\mathrm{C}$" may be absent causing so the absence of "of which C".

It is said that B is a specification of A for understand that B has all the properties of A. So, on the base of paragraphs second, third and fourth of this chapter, A_B is a specification of A if this name has a meaning. From: this; (2.1.1.3) of [1]; follows

$$Æ\langle \mathrm{A} \,/\!\!/\, \mathrm{B}\rangle \equiv \big\{\mathrm{A} \equiv \{\mathrm{A} \wedge \mathrm{B}\}\big\} \equiv \{\mathrm{A} \Rightarrow \mathrm{B}\} \tag{1.5}$$

where is intended that A is a name which has a meaning.

In relation to the following concepts of set theory reference is made to [1, 6, 7, 3, 8]. Is intended

$$\big\{\mathrm{A}_i; i = 1, \hat{\imath}\big\} \equiv \{\mathrm{A}_1, \mathrm{A}_2, \ldots, \mathrm{A}_{\hat{\imath}}\} \equiv \bigwedge_{i=1}^{\hat{\imath}} \mathrm{A}_i$$

and so a sequence and a set, both made up of $\hat{\imath}$ elements, are respectively indicated $\big(\mathrm{A}_i; i = 1, \hat{\imath}\big)$ and $\big\{\mathrm{A}_i; i = 1, \hat{\imath}\big\}$, and they differ because in the second case it is irrelevant the order defined by $\{a < b\} \equiv \{\mathrm{A}_a \text{ precedes } \mathrm{A}_b\}$ and called sequential such as the one typically own of every sequence. Therefore, a sequence is also a set but not *vice versa*. Is indicated $\{A \mid \mathcal{P}\}$ a set consisting of all the different specifications of A contextually possible when there is the condition $\mathcal{P}$. Is implicit $\{j = 1, \hat{i}\} \equiv \{j; j = 1, \hat{i}\}$.

Is meant $\mathfrak{N}\langle \underline{A} \rangle$ the numerosity of the set $\underline{A}$ i.e. the number of elements that constitute $\underline{A}$, $\text{€}\langle \underline{A} \rangle \equiv \{ş \parallel ş \in \underline{A}\}$, $\neg\underline{A}$ the set of elements that do not belong to $\underline{A}$, $\varnothing$ the empty set since $\mathfrak{N}\langle \varnothing \rangle = 0$, $\neg\varnothing$ the set constituted by each element.

The equality of the sets $\underline{A}$ and $\underline{B}$ is indicated $\underline{A} = \underline{B}$ and affirms that every element of $\underline{A}$ is also an element of $\underline{B}$ and *vice versa*. The addition of $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A} + \underline{B}$ and constituted by all the elements of $\underline{A}$ and all the elements of $\underline{B}$. The intersection of $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A} \cap \underline{B}$ and constituted by each element that belongs both to $\underline{A}$ and $\underline{B}$. The difference between $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A} - \underline{B}$ and constituted by each element of $\underline{A}$ that do not also belongs to $\underline{B}$. The union of $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A} \cup \underline{B}$ and constituted by each element that belongs to $\underline{A}$ but not to $\underline{A} \cap \underline{B}$, or to $\underline{B}$ but not to $\underline{A} \cap \underline{B}$, or to $\underline{A} \cap \underline{B}$. The Cartesian product of $\underline{A}$ and $\underline{B}$ is the set indicated $\underline{A} \times \underline{B}$ and constituted by each different pair which can be made by choosing its elements respectively belonging to $\underline{A}$ and $\underline{B}$.

These definitions, intending $\underline{A} \equiv \{A_h; h = 1, \hat{h}\}$ and $\underline{B} \equiv \{B_k; k = 1, \hat{k}\}$, are specified by

$$\begin{aligned} \{\underline{A} = \underline{B}\} &\equiv \{\mathbf{i}_{ABh} = 1; h = 1, \hat{h}\} \wedge \{\mathbf{i}_{BAk} = 1; k = 1, \hat{k}\} \\ \underline{A} \cap \underline{B} &\equiv \{\{A_h \parallel \mathbf{i}_{ABh} = 1\}; h = 1, \hat{h}\} \\ \underline{A} - \underline{B} &\equiv \{\{A_h \parallel \mathbf{i}_{ABh} = 0\}; h = 1, \hat{h}\} \\ \underline{A} \cup \underline{B} &\equiv \{\underline{A} + \underline{B}\} - \{\underline{A} \cap \underline{B}\} \end{aligned} \tag{1.6}$$

whose $\{\mathbf{i}_{ABh}; h = 1, \hat{h}\}$ is determined by the following steps (and similarly $\{\mathbf{i}_{BAk}; k = 1, \hat{k}\}$):

a) it is placed $\{\mathbf{i}_{ABh} = 0; h = 1, \hat{h}\}$;
b) they are carried out the $\mathfrak{N}\langle \underline{B} \rangle$ iterations indicated by $\{k = 1, \hat{k}\}$;
c) at the k-th iteration is searched for a $h \in \{h = 1, \hat{h}\}$ that verifies the $\{\mathbf{i}_{AB\mathrm{h}} = 0,\ A_\mathrm{h} \equiv B_k\}$ and is placed $\mathbf{i}_{AB\mathrm{h}} = 1$ if there is a such $\{\mathrm{h} \in \{h = 1, \hat{h}\} \parallel \mathbf{i}_{AB\mathrm{h}} = 0, A_\mathrm{h} \equiv B_k\}$.

A $\underline{A} \cap \underline{B} \neq \varnothing$ implies that at least one of the two sets $\{\underline{A}, \underline{B}\}$ is the addition of a subset whose elements are also elements of the other set and of another subset that does not has this property. Being then such addition and $\underline{A} \cap \underline{B} \neq \varnothing$ respective specifications of $\mathcal{P}_B$ and $\mathcal{P}_A$ in (1.3), is had a *demonstratio per absurdum* of $\underline{A} \cap \underline{B} = \varnothing$ if the addition in question must be deemed to be false because it is unjustifiable the inherent distinction between elements of a same set.

Is had

$$\{\underline{A} \subseteq \underline{B}\} \equiv \{\underline{A} = \underline{A} \cap \underline{B}\} \equiv \{\underline{B} = \underline{A} \cup \underline{B}\} \ . \tag{1.7}$$

A permutation of N elements is one of their different N! possible sequences. Is intended $\Diamond_{i=1}^{\hat{\imath}} \S_i \equiv \S_1 \diamond \S_2 \diamond \ldots \S_{\hat{\imath}}$ and

$$\left\{\Diamond, \diamond\right\} \equiv \left\{\sum, +\right\} \veebar \left\{\prod, \times\right\} \veebar \left\{\bigwedge, \wedge\right\} \veebar \left\{\bigvee, \vee\right\} \veebar \left\{\bigcap, \cap\right\} \veebar \left\{\bigcup, \cup\right\}$$

so $\Diamond_{i=1}^{\hat{\imath}} \S_i$ has the commutative property (i.e. $\Diamond_{i=1}^{\hat{\imath}} \S_i \equiv \Diamond_{i=1}^{\hat{\imath}} \S_{P_i}$ with $(P_i; i = 1, \hat{\imath})$ any one of the $\hat{\imath}!$ permutations of $(i = 1, \hat{\imath})$) and associative, with the exception of the case $\left\{\Diamond, \diamond\right\} \equiv \left\{\prod, \times\right\}$ which has the only associativity if every $\S_i$ is a set.

De Morgan's laws in propositional logic and set theory are

$$\begin{array}{ll} \neg \bigvee_{k=1}^{\hat{k}} \S_k \equiv \bigwedge_{k=1}^{\hat{k}} \neg \S_k & \neg \bigwedge_{k=1}^{\hat{k}} \S_k \equiv \bigvee_{k=1}^{\hat{k}} \neg \S_k \\ \neg \bigcup_{k=1}^{\hat{k}} \underline{A}_k \equiv \bigcap_{k=1}^{\hat{k}} \neg \underline{A}_k & \neg \bigcap_{k=1}^{\hat{k}} \underline{A}_k \equiv \bigcup_{k=1}^{\hat{k}} \neg \underline{A}_k \end{array} \tag{1.8}$$

where $\left\{\underline{A}_k; k = 1, \hat{k}\right\}$ are $\hat{k}$ sets.

Is had

$$\bigcap_{k=1}^{\hat{k}} \underline{A}_k \subseteq \bigcup_{k=1}^{\hat{k}} \underline{A}_k \qquad \bigcirc_{k=1}^{\hat{k}} \underline{A} \equiv \underline{A} \qquad \bigcirc \equiv \bigcap \veebar \bigcup \ .$$

Is intended $\dot{\bigcup}_{k=1}^{\hat{k}} \underline{A}_k$ as a $\bigcup_{k=1}^{\hat{k}} \underline{A}_k$ of which

$$\left\{\underline{A}_a \cap \underline{B}_b = \varnothing; \forall \{a, b\} \subseteq \left\{k = 1, \hat{k}\right\}\right\} ,$$

so "$\dot{\vee}$" and "$\dot{\cup}$" are specifications of the respective "$\vee$" and "$\cup$".

The Æ⟨⩒ // ∨⟩, (1.5) and the first two of (1.8) give rise to the first two of

$$\begin{array}{ll} \neg \dot{\bigvee}_{k=1}^{\hat{k}} ş_k \Rightarrow \bigwedge_{k=1}^{\hat{k}} \neg ş_k & \dot{\bigvee}_{k=1}^{\hat{k}} \neg ş_k \Rightarrow \neg \bigwedge_{k=1}^{\hat{k}} ş_k \\ \neg \dot{\bigcup}_{k=1}^{\hat{k}} \underline{A}_k \Rightarrow \bigcap_{k=1}^{\hat{k}} \neg \underline{A}_k & \dot{\bigcup}_{k=1}^{\hat{k}} \neg \underline{A}_k \Rightarrow \neg \bigcap_{k=1}^{\hat{k}} \underline{A}_k \end{array} \tag{1.9}$$

whose second two are deduced in the way obviously analogous. The $\neg ş \equiv \mathit{ş}$ and $ş \equiv \neg \mathit{ş}$ highlight how, in each of (1.8) and (1.9), $\{ş, \neg ş\}$ can be substituted by $\{\neg ş, ş\}$.

From: (1.7); $\{\underline{A} = \underline{B}\} \equiv \{\neg\underline{A} = \neg\underline{B}\}$; fourth of (1.8); (1.7); follows

$$\begin{array}{l} \{\underline{A} \subseteq \underline{B}\} \equiv \{\underline{A} = \underline{A} \cap \underline{B}\} \equiv \left\{\neg\underline{A} = \neg\{\underline{A} \cap \underline{B}\}\right\} \equiv \\ \left\{\neg\underline{A} = \{\neg\underline{A} \cup \neg\underline{B}\}\right\} \equiv \{\neg\underline{B} \subseteq \neg\underline{A}\} \; . \end{array} \tag{1.10}$$

A univocal (i.e. non-injective and surjective) correspondence between $\underline{A}$ and $\underline{B}$ is a set of $\mathfrak{N}_{\underline{A}}$ pairs indicated $\underline{A} \twoheadrightarrow \underline{B}$ and defined by a $\underline{A} \twoheadrightarrow \underline{B} \equiv \left\{A_h, B_{k_h}; h = 1, \hat{h}\right\}$ of which

$$\left\{k_h \in \{k = 1, \hat{k}\}; h = 1, \hat{h}\right\} \qquad \left\{k \in \{k_h; h = 1, \hat{h}\}; k = 1, \hat{k}\right\} \; .$$

Therefore a $\underline{A} \twoheadrightarrow \underline{B}$ makes to correspond to each $\in\langle\underline{A}\rangle$ a only $\in\langle\underline{B}\rangle$ and in such $\underline{A} \twoheadrightarrow \underline{B}$ appear all the elements of $\underline{A}$ and $\underline{B}$.

A bijection, i.e. a biunivocal correspondence, i.e. a one-to-one (injective) and onto (surjective) correspondence, between $\underline{A}$ and $\underline{B}$ of which $\mathfrak{N}_{\underline{A}} = \mathfrak{N}_{\underline{B}}$, is a set of $\mathfrak{N}_{\underline{A}}$ pairs indicated $\underline{A} \twoheadleftarrow\!\!\twoheadrightarrow \underline{B}$ and defined by a

$$\underline{A} \twoheadleftarrow\!\!\twoheadrightarrow \underline{B} \equiv \left\{A_h, B_{k_h}; h = 1, \hat{h}\right\} \quad \text{of which} \quad \left\{k_h; h = 1, \hat{h}\right\} = \left\{k = 1, \hat{k}\right\} \; .$$

Therefore a such $\underline{A} \twoheadleftarrow\!\!\twoheadrightarrow \underline{B}$ makes to correspond to each $\in\langle\underline{A}\rangle$ a only $\in\langle\underline{B}\rangle$ and *vice versa.*

In the following are treated (with reference to [9, 10]) dispositions, permutations and combinations "simple" i.e. "without repetitions". A disposition of class K of N objects is a sequence of K elements of a set consisting of N elements, so two dispositions may also differ only for the respective sequential orders. Instead a combination of class K of N objects is a subset of numerosity K of a set of numerosity N, so the sequential order of the elements of a combination is irrelevant as in the case of the sets. A disposition of class N of N objects is called also permutation, and a disposition of class K of N objects is also called permutation of N objects taken K at a time. The respective number of all the possible different dispositions and combinations of class K of N objects is $\frac{N!}{(N-K)!}$ and $\binom{N}{K}$ with the second (the binomial coefficient) of which $\binom{N}{K} \equiv \frac{N!}{K!(N-K)!}$.

Calling $\underset{\sim}{k}_{cba}$ the a-th element of the b-th different combination of class c of the $\{k=1,\hat{\mathrm{k}}\}$, is had

$$\left\{\left\{\underset{\sim}{k}_{cba}; a=1,c\right\}; b=1,\binom{\hat{\mathrm{k}}}{c}\right\} \Leftrightarrow$$
$$\left\{\left\{k=1,\hat{\mathrm{k}}\right\}-\left\{\underset{\sim}{k}_{cba}; a=1,c\right\}; b=1,\binom{\hat{\mathrm{k}}}{c}\right\} \equiv \tag{1.11}$$
$$\left\{\left\{\underset{\sim}{\mathrm{k}}_{cba}; a=1,\hat{\mathrm{k}}-c\right\}; b=1,\binom{\hat{\mathrm{k}}}{\hat{\mathrm{k}}-c}\right\} .$$

The (2.2.36) and (2.2.37) of [1] affirm the respective

$$\mathfrak{N}\left\langle\bigcup_{k=1}^{\hat{\mathrm{k}}} \underline{\mathrm{A}}_k\right\rangle = \sum_{c=1}^{\hat{\mathrm{k}}}(-1)^{c+1}\sum_{b=1}^{\binom{\hat{\mathrm{k}}}{c}} \mathfrak{N}\left\langle\bigcap_{a=1}^{c} \underline{\mathrm{A}}_{\underset{\sim}{k}_{cba}}\right\rangle$$
$$\mathfrak{N}\left\langle\dot{\bigcup_{k=1}^{\hat{\mathrm{k}}}} \underline{\mathrm{A}}_k\right\rangle = \sum_{k=1}^{\hat{\mathrm{k}}} \mathfrak{N}\langle\underline{\mathrm{A}}_k\rangle \ . \tag{1.12}$$

The $\bigsqcup_{k=1}^{\hat{\mathrm{k}}}\underline{\mathrm{A}} \equiv \underline{\mathrm{A}}$ entails $\mathfrak{N}\left\langle\bigsqcup_{k=1}^{\hat{\mathrm{k}}}\underline{\mathrm{A}}\right\rangle = \mathfrak{N}_{\underline{\mathrm{A}}}$ which is coherent with the first of (1.12) and the verify

$$\sum_{c=1}^{\hat{\mathrm{k}}}(-1)^{c+1}\sum_{b=1}^{\binom{\hat{\mathrm{k}}}{c}} 1 = \sum_{c=1}^{\hat{\mathrm{k}}}(-1)^{c+1}\binom{\hat{\mathrm{k}}}{c} = 1 \ .$$

Inherently the $\hat{\mathrm{h}}\hat{\mathrm{k}}$ sets $\left\{\underline{\mathcal{A}}_{hk}; h=1,\hat{\mathrm{h}}; k=1,\hat{\mathrm{k}}\right\}$, in section 2.2 of [1] is had

$$\bigcap_{h=1}^{\hat{\mathrm{h}}}\prod_{k=1}^{\hat{\mathrm{k}}} \underline{\mathcal{A}}_{hk} \supseteq \prod_{k=1}^{\hat{\mathrm{k}}}\bigcap_{h=1}^{\hat{\mathrm{h}}} \underline{\mathcal{A}}_{hk}$$

and (2.2.30) i.e.

$$\neg\exists\left\{\begin{array}{l}\{ş \mid ş \equiv ş; ş \in \underline{A}_{ac}\} \neq \{ş \mid ş \equiv ş; ş \in \underline{A}_{bc}\}\,; \\ ş \in \underline{A}_{ac}; ş \in \underline{A}_{bc}; \{a,b\} \subseteq \left\{h = 1, \hat{\mathrm{h}}\right\}; \\ c \in \left\{k = 1, \hat{\mathrm{k}}\right\}\end{array}\right\} \Rightarrow \left\{\bigcap_{h=1}^{\hat{\mathrm{h}}} \prod_{k=1}^{\hat{\mathrm{k}}} \underline{A}_{hk} = \prod_{k=1}^{\hat{\mathrm{k}}} \bigcap_{h=1}^{\hat{\mathrm{h}}} \underline{A}_{hk}\right\} \tag{1.13}$$

which both can result from computer verifications if each $\underline{A}_{hk}$ is a finite set, while (1.13) can result by representing the products as rectangular parallelepipeds $\hat{\mathrm{k}}$-dimensional if each $\underline{A}_{hk}$ is an interval of real numbers.