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# Algebraic Quantum Theory of Consciousness

A Solution to the Problem of Quantum Collapse
in Systems Having Three Anticommuting Elements

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ISBN 978-88-255-1707-1



I edition: October 2018

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*Algebraic Quantum Theory of Consciousness*
ISBN 978-88-255-1707-1
DOI 10.4399/97888255170711
pag. 7–9 (october 2018)

# Preface

by Antonio Di Ieva*

If it is true that the great book of nature is written in mathematical language, as quoted by Galileo Galilei, it is also true that even the comprehension of the most complex and sublime natural product, i.e. the human brain, cannot neglect the use of mathematics, without which "we would wander in vain through a dark labyrinth". This has been well understood by several mathematicians and scientists, and in such a tradition, Elio Conte et al. have brought the hard problem of consciousness to higher and unexplored peaks: The interpretation of the mind by means of the language of arithmetic, specifically, of the Clifford algebra.

Far from philosophical speculations on the monadic nature of numbers, Conte follows a heuristic approach to describe the algebra of consciousness, as he speculates that the elements of the Clifford algebra may represent the basic entities of the mind. The foundation of the theory lies on the architecture of the brain itself, in which neuronal circuitries involved in the use and knowledge of numbers, as well as in calculations, have a specific anatomical substrate, that is quite spread but different from the domains of other functions (e.g. language). Indeed language skills are not necessary for computation, as shown in patients with autism and/or some mathematical genius, as well as in the known dyscalculia, which affects patients with damage in specific cerebral networks. In a kind of unified theory, Conte is able to link the problem of the consciousness to the theory of complexity, overcoming centuries of philosophical and mathematical speculations.

The search of the seat of the soul, of the consciousness, has been unsuccessful over the last centuries of philosophical and scientific investigations, and the omnipresent Cartesian dualism between body

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and mind has delayed the use of a more comprehensive approach to understand the problem of the mind. When the brain is investigated in relationship to the consciousness, cognitive neuroscientists and clinicians are aware of the fact that the whole is greater than the sum of its parts. Despite this, Conte uses a reductionist approach based on different tiers of analysis, from the quantum physics of the microstructural bricks of the brain to the complexity of consciousness generated in a regime of chaotic, fractal and multifractal mechanisms, in which the fractal dynamics creates a recursive system of auto–referential variables, isomorphic structures and dynamics across scales.

Sailing in the stormy ocean of the conceptual grounds expressed by Chomsky, Clifford, von Neumann, Pinker, and others, Conte uses a quantitative approach ("measurements as semantic acts") aimed to quantify an abstract concept, i.e., the consciousness, in what he calls the "Mind Entity". The use of axioms is as precise as a surgical incision, aimed to dissect the mind entity into neuropsychological systems, that may be viewed, in his weltanschauung, as agents mirroring themselves, in a kind of self referential system, reminiscent of an artistic circle in some of the Escher's drawings.

In the first level of analysis of this book, the "nuts and bolts" of Clifford algebra and its application to the theory of mind and neurosciences are introduced. It has to be clear that it is not an easy stroll in the woods of narrative. It is a very steep pathway used by Conte to give solid bases to his hypotheses, which can be truly appreciated only with a deep mathematical background. Subsequently the consciousness is characterized by means of quantum mechanics, and the reader is brought into the maelstrom of Pauli matrices and Hilbert space. Then the explorer of these hostile lands is introduced into the failure of what the American philosopher and cognitive scientist Daniel Dennett derisively defined as the "Cartesian Theatre", in which the res cogitans and res extensa are supposed to meet each other somewhere in the brain to merge into the consciousness.

Conte overcomes such Cartesian dualism, merging the two entities in the quantum mechanics model, mainly focused on the well–known phenomenon of entanglement: « mental entities relating semantic acts, ideas, cognition, may be entangled ».

At this point, proceeding with the reading of the book the reader is catapulted into the central core of the paradigm, in which it is shown that consciousness is intrinsically connected to the quantum spin. Not a novel idea, as already considered by the physicist Roger Penrose and anesthesiologist and psychologist Stuart Hameroff in their merged Orchestrated Objective Reduction hypothesis, which contemplates the idea that quantum activities at the level of the intraneuronal microtubules give rise to the emergence of the consciousness. But Conte is able to mathematically link neurology, on the basis of previous speculations by Huping Hu et al., to the quantum approach, in which the spin chemistry of the oxygen atom and the free radical nitric oxide (which is also a neurotransmitter!) is the primum movens of the consciousness. By this way, as suggested also by other authors, the spin–related theory overcomes even the lipid and the protein theories to explain states of alteration and/or temporary suspension of consciousness, e.g., during neuroanesthesia. To close the argumentations' loop, the neuronal action potentials are seen as modulators of the spin–network, rather than excitatotory/inhibitors of actions, giving rise to the consciousness, which « emerges from the collapses of those entangled quantum states ».

In the holistic phenomenology of the book, some heuristic argumentations are discussed as well, such as the lack of perturbation (or at least, the minimal consciousness perturbations) occurring in subjects undergoing high- or even ultra–high field Magnetic Resonance Imaging (e.g., 7 Tesla MRI), in which the spin alterations should, in theory, strongly affect the consciousness. Some speculations are reported, such as "even strong static magnetic fields only have small effects on the thermal dynamics of the neural nuclear spin ensembles" and "The brain has likely developed other mechanisms through evolution to counter the effects of most external disturbances". Although the Occam's razor is required to confirm or refute such speculations by means of further experimental studies, it remains incontrovertible the fact that this book offers a fascinating introduction into the unexplored universe of the algebraic quantum theory of consciousness.

*Algebraic Quantum Theory of Consciousness*
ISBN 978-88-255-1707-1
DOI 10.4399/97888255170712
pag. 11–43 (october 2018)

# A Bare Bone Skeleton of Quantum Mechanics

## The Solution of Quantum Collapse Problem in a System of Three Anticommuting Elements and Identification of the Quantum Algebra of Consciousness

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Abstract

After an articulated exposition of the basic features of the Clifford algebra we give evidence that the basic elements of this algebra may represent the basic entities of the mind. According to the previous basic results of V.A. Lefebvre on conscience, we also delineate some peculiarities of the consciousness and we give proof that they may be correctly represented by this algebra.

## 1. A Bare Bone Skeleton of Quantum Mechanics

Consciousness is an abstract identity marked from several and unique features but mainly is marked from two basic salient and peculiar properties.

*a*) It is an entity that has self–awareness and this is to say that it has in its inner the image of itself. In most cases we speak of self–image to represent such peculiar feature.
*b*) It has awareness of an external space–time located abstract entity.

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Every one is convinced that it is extremely difficult to conceive and to represent a system having such self–referential properties and this is the reason because from several and several years of activity we mark the problem to represent the consciousness as the basic "hard problem" of science.

Constantly the hard problem involves the interest of theoretical physics, biological, medical and, in general, Life Sciences. It is a basic problem in neurology as well as in science of mind here including the tentative to approach this problem under the field of the philosophy. Physicists usually approach the problem from a particular perspective: may we use the basic foundations of physics to explain the two previous mentioned peculiar features of consciousness? In detail, do we have the mathematical instruments which, pertaining to physical formulation, are able to approach the hard problem providing an explanation of the previous mentioned basic features?

The list of physicists who have engaged in this hard problem is of course endless but we acknowledge, in particular, one scholar who in years of activity has conducted studies and has given us some fundamental indications and results. This scientist is V.A. Lefebvre who, in fact, has culminated his activity with a celebrated book entitled *The Algebra of the Conscience*. Here, we consider the algebra of consciousness: a tentative to indicate that consciousness may be described by algebra and thus by a mathematical tool.

This is precisely the question that we attempt to develop in this work, to describe this algebra for the first time, able to delineate the two basic peculiar features of consciousness that we have previously indicated.

Let us start to present this algebra.

Let us start with a proper definition of the 3–D space Clifford (geometric) algebra $Cl_3$.

It is an associative algebra generated by three vectors $e_1$, $e_2$, and $e_3$ that satisfy the orthonormality relation

$$e_j e_k + e_k e_j = 2\delta_{jk} \text{ for } j, k, \lambda \in [1, 2, 3]. \tag{1}$$

That is,

$$e_\lambda^2 = 1 \text{ and } e_j e_k = -e_k e_j \text{ for } j \neq k.$$

Let a and b be two vectors spanned by the three unit spatial vectors in $Cl_{3,0}$. By the orthonormality relation the product of these two vectors is given by the well known identity: ab $= a \cdot b + i(a \times b)$ where $i = e_1 e_2 e_3$ is a Clifford algebraic representation of the imaginary unity that commutes with vectors.

The (1.1) are well known in quantum mechanics. Here, we present a proof under an algebraic profile. Let us follow the approach that, starting with 1981, was developed by Y. Ilamed and N. Salingaros [1]. Imaeda and Edmonds also used this algebra extensively in the past [2, 3].

Let us consider that three abstract basic elements, $e_i$, with $i = 1, 2, 3$, admit the following two postulates:

*a*) it exists the scalar square for each basic element:

$$e_1 e_1 = k_1,\ e_2 e_2 = k_2,\ e_3 e_3 = k_3 \text{ with } k_i \in \Re. \tag{2}$$

In particular we have also the unit element, $e_0$, such that that

$$e_0 e_0 = 1, \text{ and } e_0 e_i = e_i e_0;$$

*b*) the basic elements $e_i$ are anticommuting elements, that is to say:

$$e_1 e_2 = -e_2 e_1,\ e_2 e_3 = -e_3 e_2,\ e_3 e_1 = -e_1 e_3. \tag{3}$$

*Theorem n. 1*

Assuming the two postulates given in (a) and (b) with $k_i = 1$, the following commutation relations hold for such algebra:

$$\begin{aligned} &e_1 e_2 = -e_2 e_1 = i e_3; \quad && e_2 e_3 = -e_3 e_2 = i e_1; \\ &e_3 e_1 = -e_1 e_3 = i e_2; \quad && i = e_1 e_2 e_3, (e_1^2 = e_2^2 = e_3^2 = 1) \end{aligned} \tag{4}$$

They characterize the Clifford *Si* algebra. We will call it the algebra A(Si).

Proof.

Consider the general multiplication of the three basic elements $e_1, e_2, e_3$, using scalar coefficients $\omega_k, \lambda_k, \gamma_k$ pertaining to some field:

$$\begin{aligned} &e_1e_2 = \omega_1 e_1 + \omega_2 e_2 + \omega_3 e_3; \quad e_2e_3 = \lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3; \\ &e_3e_1 = \gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3. \end{aligned} \tag{4a}$$

Let us introduce left and right alternation: for any $(i, j)$, associativity exists $e_ie_ie_j = (e_ie_i)e_j$ and $e_ie_je_j = e_i(e_je_j)$ that is to say

$$\begin{aligned} e_1e_1e_2 &= (e_1e_1)e_2; \quad e_1e_2e_2 = e_1(e_2e_2); \\ e_2e_2e_3 &= (e_2e_2)e_3; \quad e_2e_3e_3 = e_2(e_3e_3); \\ e_3e_3e_1 &= (e_3e_3)e_1; \quad e_3e_1e_1 = e_3(e_1e_1). \end{aligned} \tag{5}$$

Using the (4) in the (5), it is obtained that

$$\begin{aligned} &k_1e_2 = \omega_1 k_1 + \omega_2 e_1e_2 + \omega_3 e_1e_3; \quad k_2e_1 = \omega_1 e_1e_2 + \omega_2 k_2 + \omega_3 e_3e_2; \\ &k_2e_3 = \lambda_1 e_2e_1 + \lambda_2 k_2 + \lambda_3 e_2e_3; \quad k_3e_2 = \lambda_1 e_1e_3 + \lambda_2 e_2e_3 + \lambda_3 k_3; \\ &k_3e_1 = \gamma_1 e_3e_1 + \gamma_2 e_3e_2 + \gamma_3 k_3; \quad k_1e_3 = \gamma_1 k_1 + \gamma_2 e_2e_1 + \gamma_3 e_3e_1. \end{aligned} \tag{6}$$

From the (6), using the assumption (b), we obtain that

$$\begin{aligned} &\tfrac{\omega_1}{k_2} e_1e_2 + \omega_2 - \tfrac{\omega_3}{k_2} e_2e_3 = \tfrac{\gamma_1}{k_3} e_3e_1 - \tfrac{\gamma_2}{k_3} e_2e_3 + \gamma_3; \\ &\omega_1 + \tfrac{\omega_2}{k_1} e_1e_2 - \tfrac{\omega_3}{k_1} e_3e_1 = -\tfrac{\lambda_1}{k_3} e_3e_1 + \tfrac{\lambda_2}{k_3} e_2e_3 + \lambda_3; \\ &\gamma_1 - \tfrac{\gamma_2}{k_1} e_1e_2 + \tfrac{\gamma_3}{k_1} e_3e_1 = -\tfrac{\lambda_1}{k_2} e_1e_2 + \lambda_2 + \tfrac{\lambda_3}{k_2} e_2e_3 \end{aligned} \tag{7}$$

We have that it must be

$$\omega_1 = \omega_2 = \lambda_2 = \lambda_3 = \gamma_1 = \gamma_3 = 0 \tag{8}$$

and

$$\begin{aligned} &-\lambda_1 k_1 + \gamma_2 k_2 = 0 \quad \gamma_2 k_2 - \omega_3 k_3 = 0 \\ &\lambda_1 k_1 - \omega_3 k_3 = 0 \end{aligned} \tag{9}$$

The following set of solutions is given:

$$k_1 = -\gamma_2 \omega_3, \quad k_2 = -\lambda_1 \omega_3, \quad k_3 = -\lambda_1 \gamma_2 \tag{10}$$

that is to say

$$\omega_3 = \lambda_1 = \gamma_2 = i. \tag{11}$$

In this manner, as a theorem, the existence of such algebra is proven. The basic features of this algebra are given in the following manner

$$\begin{aligned}
&e_1^2 = e_2^2 = e_3^2 = 1; \qquad e_1 e_2 = -e_2 e_1 = i e_3;\\
&e_2 e_3 = -e_3 e_2 = i e_1; \quad e_3 e_1 = -e_1 e_3 = i e_2;\\
&i = e_1 e_2 e_3.
\end{aligned} \tag{12}$$

The content of the theorem n.1 is thus established: given three abstract basic elements as defined in (a) and (b) ($k_i = 1$), an algebraic structure is given with four generators ($e_0, e_1, e_2, e_3$).

Note that in the algebra *A(Si)* the $e_i (i = 1, 2, 3)$ have an intrinsic potentiality that is to say an ontic potentiality or equivalently an irreducible intrinsic indetermination. Since $e_i^2 = 1 (i = 1, 2, 3)$, the numerical value +1 or the numerical value –1 are potentially possible. Such two alternatives (+1 and -1) both coexist ontologically and this potential possibility intrinsically travels in each possible formal application of this algebra.

A generic member of our algebra *A(Si)* is given by

$$x = \sum_{i=0}^{4} x_i e_i = x_0 + \boldsymbol{x} \tag{13}$$

with $x_i$ pertaining to some field $\Re$ or $C$.

We may define [2] the hyperconjugate $\hat{x}$

$$\hat{x} = x_0 - \boldsymbol{x}$$

the complex conjugate

$$x* = x_0^* + \boldsymbol{x}^\circ$$

and the conjugate

$$\bar{x} = x_0^* - \boldsymbol{x}^*.$$

The *Norm* of x is defined as

$$Norm\ (x) = x\,\hat{x} \ = \ \hat{x}\,x \ = \ x_0^2 - x_1^2 - x_2^2 - x_3^2 \tag{14}$$

with

$$Norm\ (xy) = Norm\ (x)\ Norm\ (y).$$

The proper inverses of the basic elements $e_i(i = 1, 2, 3)$ are themselves. Given the member x, its inverse $x^{-1}$

$$\text{is } \hat{x}/Norm\ (x) \text{ with } Norm\ (x) \neq 0.$$

We may transform Clifford members according to Linear Transformations

$$x' = AxB + C \tag{14a}$$

with unitary norms for the employed Clifford members $A, B$ and $C = 0$ for linear homogeneous transformation.

Let us now take a step on.

As previously said, in the algebra *A(Si)* the $e_i(i = 1, 2, 3)$ have an intrinsic potentiality, an ontic potentiality or equivalently an irreducible intrinsic indetermination. Let us give proof of such our basic assumption.

Since the $e_i$ are abstract entities, having the potentiality (+1,-1), they have an intrinsic and irreducible indetermination. Therefore, we admit to be $p_1(+1)$ the probability that $e_1$ relates the value (+1) and $p_1(-1)$ the probability that relates the value $-1$. We may represent the mean value that is given by

$$< e_1 >= (+1)p_1(+1) + (-1)p_1(-1). \tag{15}$$

Considering the same corresponding notation for the two remaining basic elements, we may introduce the other two mean values:

$$\begin{aligned} < e_2 > &= (+1)p_2(+1) + (-1)p_2(-1), \\ < e_3 > &= (+1)p_3(+1) + (-1)p_3(-1). \end{aligned} \tag{16}$$

We have

$$-1 \leq < e_i > \leq +1 \quad i = (1, 2, 3). \tag{17}$$

We select now the following generic element of the algebra *A(Si)*:

$$x = \sum_{i=1}^{3} x_i e_i \quad x_i \in \Re. \tag{18}$$

Note that

$$x^2 = x_1^2 + x_2^2 + x_3^2. \tag{19}$$

Its mean value results to be

$$< x > = x_1 < e_1 > + x_2 < e_2 > + x_3 < e_3 > . \tag{20}$$

Let us call

$$b = (x_1^2 + x_2^2 + x_3^2)^{1/2} \tag{21}$$

so that we may attribute to $x$ the value $+b$ or $-b$.
We have that

$$-b \leq x_1 < e_1 > + x_2 < e_2 > + x_3 < e_3 > \leq b. \tag{22}$$

The (22) must hold for any real number $x_i$, and, in particular, for

$$x_i = < e_i >$$

so that we have that

$$x_1^2 + x_2^2 + x_3^2 \leq b$$

that is to say

$$b^2 \leq b \rightarrow b \leq 1$$

so that we have the fundamental relation

$$< e_1 >^2 + < e_2 >^2 + < e_3 >^2 \leq 1. \tag{23}$$

These results were also previously obtained by Jordan but not using Clifford algebra [4]. Our results are contained in [5–34] where we mention also the contributions of also authors that inspired our work as in particular Altafini, Orlov, Cini.

This is a basic relation of irreducible indetermination that we are writing in our Clifford algebraic elaboration.

Let us observe some important features:

*a*) in absence of a direct numerical attribution to the $e_i$ (and in analogy with physics this means... in absence of a measurement, that is to say in absence of direct observation or thinking about one such quantum observable), the (23) holds. If we attribute instead a definite numerical value to one of the three entities because we have performed a direct measurement with recording the resulting value of the instrument, as example we have for $e_3$ the numerical value +1, we have $< e_3 >=1$, and the (23) is reduced to

$$< e_1 >^2 + < e_2 >^2 = 0, \quad < e_1 > = < e_2 > = 0, \tag{24}$$

and we have complete, irreducible, indetermination for $e_1$ and for $e_2$;

*b*) finally, the (23) affirms that we can never simultaneously attribute definite numerical values to two basic non commutative elements $e_i$.

We may now summarize the obtained results.

First, we retain that the first axiom of the *A(Si)* algebra, the (2) with $k_i = 1$, indicates that the abstract basic elements $e_i$ have an ontic potentiality. This is to say that they have an irreducible indeterminism as supported finally from the (23). In order to characterize such features we have used the concept of mean value for such algebraic entities and that one of potentiality. When, thought an instrument of measurement, we attempt to attribute a numerical value to an abstract element, as it happens as example in the (24), we perform an operation that in physics has a counterpart that is called an act of measurement. For us, any measurement is a semantic act, no matter if the measurement is performed by a technical instrument or by human observer. In any case, it is realized having in its basic structure, a semantic act. Note that we are using the abstract space of the $e_i$ elements in *A(Si)*. Therefore, also thinking, a cognitive or a semantic act to attribute a numerical value, is a measurement.

Relating this last feature, remaining in the restricted domain of the *A(Si)* algebra, we are, in some sense in a condition that, on the general plane, may be assimilated to that one in which we have human or technical systems that are in some manner forced to answer to questions (the attribution of numerical values to the basic elements) which they cannot understand in line of principle in a definite manner. As consequence, the probabilities that we have used in the (15) and in the (16) are fundamentally different from classical probabilities under a basic conceptual profile. According to the classical probability theory, as it is known, probabilities represent a lack of information about preexisting and pre–established properties of systems. In the present case, we have instead a situation in which we do not have an algorithm in *A(Si)* to execute a semantic act devoted to identify with certainty the meaning of a statement in terms of truth values and in relation to another statement. Here, only probabilities on the true value exist and they pertain now not to a missing our knowledge but to basic intrinsic foundation of irreducible indetermination in the inner structure of our reality.

## 2. A Theoretical Elaboration Using Clifford Algebra

Let us evidence another important feature of Clifford algebra *A(Si)*.

In Clifford algebra *A(Si)* we have idempotents (as counterpart we have projection operators in quantum mechanics). In von Neumann language projection operators can be interpreted as logical statements.

Let us give some example of idempotents in Clifford algebra.

The central role of density matrix in traditional quantum mechanics is well known. In the Clifford algebraic scheme, we have a corresponding algebraic member that is given in the following manner

$$\rho = a + be_1 + ce_2 + de_3 \tag{25}$$

with

$$a = \frac{|c_1|^2}{2} + \frac{|c_2|^2}{2}, \quad b = \frac{c_1^* c_2 + c_1 c_2^*}{2}, \quad c = \frac{i(c_1 c_2^* - c_1^* c_2)}{2}, \quad d = \frac{|c_1|^2 - |c_2|^2}{2} \tag{26}$$

where the $e_i$ are the basic elements in the algebraic Clifford scheme while in matrix notation, $e_1$, $e_2$, and $e_3$ in standard quantum mechanics are the well known Pauli matrices. The complex coefficients $c_i(i = 1, 2)$ are the well known probability amplitudes for the considered quantum state

$$\psi = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \text{ and } |c_1|^2 + |c_2|^2 = 1. \tag{27}$$

For a pure state in quantum mechanics, it is $\rho^2 = \rho$. In our scheme we have demonstrated a theorem in Clifford algebra as following:

$$\rho^2 = \rho \longleftrightarrow a = \frac{1}{2} \text{ and } a^2 = b^2 + c^2 + d^2. \tag{28}$$

The details of this theorem of ours are given in references. We have also $Tr(\rho) = 2a = 1$. In this manner, we have the necessary and sufficient conditions for $\rho$ to represent a Clifford member whose counterpart in standard quantum mechanics represents a potential state or, equivalently, a superposition of pure states.

Let us now consider the two other of such idempotents in *A(Si)*

$$\psi_1 = \frac{1 + e_3}{2} \text{ and } \psi_2 = \frac{1 - e_3}{2}. \tag{29}$$

It is easy to verify that $\psi_1^2 = \psi_1$ and $\psi_2^2 = \psi_2$.

Let us examine now the following algebraic relations:

$$e_3\psi_1 = \psi_1 e_3 = \psi_1 \tag{30}$$

$$e_3\psi_2 = \psi_2 e_3 = -\psi_2. \tag{31}$$

Similar relations hold in the case of $e_1$ or $e_2$.

Here is one interesting feature. By a pure semantic act, looking at the (30) and (31), we reach only a conclusion. With reference to the idempotent $\psi_1$, the algebra *A(Si)* (see the (30)), attributes to $e_3$ the numerical value of +1 while, with reference to the idempotent $\psi_2$, the algebra *A(Si)* attributes to $e_3$(see the (31)), the numerical value of –1.

The basic point is that at the basis we have here only a semantic act.