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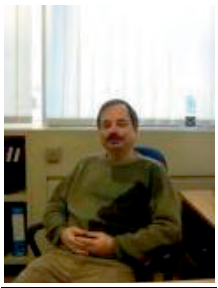

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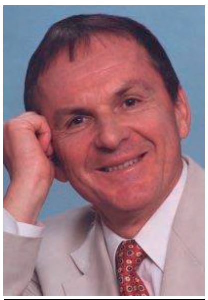

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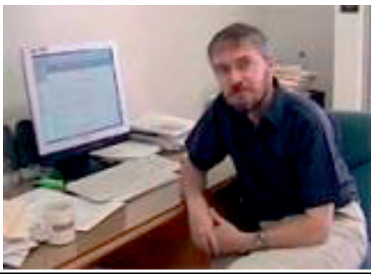

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He Lived Here
In memory of Stephen Hawking

Oxford, 8 January 1942 – Cambridge, 14 March 2018

What a strange awakening today. Stephen Hawking – escaped from an infaust diagnosis 21 years ago and from many fatal surgeries- is gone.

My generation has grown up with catchphrases such as "where were you when John Lennon died?" I think that Stephen Hawking death will bring back similar questions in the future. As it has been said for Einstein: "He lived here". I'm not speaking only of the powerful and empathetic relation he and his work had with the media and audience. All of us admired Eddie Redmayne in *The Theory of Everything* and, earlier, Benedict Cumberbutch playing the role of Hawking, all of us know something about black holes and their radiation; and *A Brief History of Time* is surely one of the the most successful book of all time.
Actually, there another reason why the Stephen Hawking death will be stuck in our minds. Just for once, the image in the media was *really* the man behind and beyond the news. You could always perceive he was a man of stature and an intense person, there was something unique between the brightening of his eyes and the lines of his most technical papers. Einstein used to say that a theoretical physicist can appear to be an opportunist with no scruples to epistemologists. The reason lies in the fact that a scientist uses precise tools, mathematics for theoretical physicists, and not the power of interpretations; in no way a scientist worries about giving a frame to a result so mimicking a philosopher. That's where Hawking was, maybe, the most secular among the scientists. He never married a theory, but he wooed them all, just like he never failed to smile at a pretty woman. He always questioned how far a theory could we lead, and we could *actually* say about the Universe. Among all the things that gave him everlasting fame there are

Fig. 1 Stephen Hawking with W.J. Kaufmann (1977)

two problems which really stay at the extreme borders of knowledge. One deals with the final stage of massive stars, the famous Black Holes, which, according to Einstein, during the last stage of their lives should enter on infinite collapse, a *singularity.* Laplace had already saw it, as well as Oppenheimer and Landau later, up to Wheeler and his master Sciama. Nobody had ever investigated it before as Hawking did. Singularity had wandered like a monster in specialized reviews and journals for some years, later – thanks to Hawking and Penrose – it became clear that it was just a structural limitation of Einstein gravitation theory and it was time to give room to a new theory, the *quantum gravity*, which is still a frontline topic in theoretical physics. Stephen Hawking was the one who reached some milestones in this new field, the black hole radiation and the information analysis of a physical system with Hawking-Bekenstein formula. Black holes were just an exercise, because now Stephen was ready to look at the more mysterious singularity, the *Big Bang.* It was about in the '80s, he and Jim Hartle proposed the *no-boundary Universe*, a charming expression that we can coarsley translate by saying that space and time emerge from a *quantum nebulosity*; something similar to the Nicola Cusano Universe, where there is no before and no after, where each point is the center. Or, just to be a bit more technical, where time is *curved and imaginary* before collapsing into what we see and what the Standard Model describes.

Similarly to all the other physicists, I happen to quote Stephen thousand times, and every time it was an occasion to read his works again. I admired his ability to build an apparently impenetrable castle of mathematics all around a strong idea. He could have been an excellent chess player. I say "apparently" because Hawking knew very well that mathematics was a sublime form of rethoric which could always be attacked or taken apart. Or started from scratch. What really makes the difference for a physical theory are generalities and the steadiness of its starting points. Sometimes, a weak point could be found in Stephen's approach (*shrewd, very subtle!*), but , at the same time, you couldn't help but notice how the question had been posed with absolute clearness and how it would be really difficult to do it better. Leonardo Chiatti and I started from Hartle-Hawking theory to develop the idea of the Archaic Universe [1,2,3]and, recently, Fabrizio Tamburini, Maria Felicia de Laurentis and I have discovered a particular mode of Hawking radiation, the so-called *soft hairs.* There's only a case when Hawking admitted to be defeated, in front of a young Don Page, about the end of the Universe, namely about the possibility that the whole wave-function rewinds to go back to origins. Like it happened some years before with Kip Thorne about the possibility to discover a black hole in Cygnus X-1, also in that occasion a stake was paid: a magazine subscription (in the case of Kip, it was a yearly subscription to *Playboy*). In my opinion, the Hawking idea is well-grounded, so the last word has not been spoken.

Maybe, the most don't know that there is a beautiful theatrical play titled *God and Stephen Hawking* on Stephen Hawking life and his struggles with his disease and the biggest mysteries of the Universe. The author, Robin Hawdon, was really within "the zone" when wrote it, you can find in the play the same humor which has became the irreducible trait of Stephen.

In the end there are some cues echoing the closing lines of *A Brief History of Time,* where Stephen reaffirms his faith in a Final Reason, it equates him with giants like Einstein. Let's listen to it once again:
STEPHEN: I do know it is there, inherent to the infinite experiment of the Universe. A solution that – differently from any metaphysical theory and belief – will look to be so clear....so patent...and we will realize it has been with us all the time.
Bye Stephen!

Ignazio Licata

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Ignazio Licata

Preface

In the first quarter of 2018, we present a collection of fourteen manuscripts covering important topics of theoretical and mathematical physics ranging from quantum walk, gravitational waves, string theory, gauge field theories and canonical formalism, gravitational thermodynamics and quantum gravity, neutrino masses and effective Majorana, relativistic Klein-Gordan equation, thermodynamics of hot Quantum scalar field, Spin and Zitterbewegung, solutions to the gravitational field equations in curved phase-spaces, hadron mass quantization, Neimark-Sacker bifurcation and chaotic attractors for discrete dynamical systems, and Dirac space in the Quantum relativistic theory.
Lanéry on his paper presents a self-contained introduction of the projective limits of state spaces: quantum field theory without a vacuum and its relations to other QFT approaches. Mansour et al. addresses in his paper the Faddeev-Jackiw quantization methodology in the noncommutative structure of massive Bosonic strings. Margolin defined the gravitational thermodynamics for minimal length and minimal inverse temperature. Kaminaga in paper propose the Poisson bracket for a new canonical theory. Damanik in his work derives a neutrino mass matrix from cobimaximal neutrino mixing matrix in parallel with effective Majorana mass. Debnath address relativistic Klein-Gordan equation for q-deformed modified Eckart plus Hylleraas potential. Jafarizadeh et al. on their work on graph isomorphism problem investigate Fermionic quantum walk for detecting Nonisomorph Cospectral Graphs. Rojas et al. use the brick wall model to calculate of free energy of quantum scalar field in a curved spacetime (D +1) dimensions. Recami et al. in his paper "Spin and Zitterbewegung" address the classical theory of the electron in parallel with quantum analogue to extend a new non-linear Dirac-like equation. Castro in his paper gives mathematical solutions to the gravitational field equations in curved phase-spaces. Fathi presents dialectic transformation media within gravitational waves. Hothi et al. show the validation of the Hadron mass quantization from experimental Hadronic Regge trajectories. Yahiaoui et al. in their cryptographic work discuss the dynamics and bifurcations of a family of two-dimensional noninvertible maps. Temnenko in his 4th paper of the series of physics of currents and potentials addresses Dirac space and vectors.
I want to express my sincere gratitude to the my friend *Ignazio Licata* for the valuable discussions, reviewing and excellent editorial work, and thanks to my friend *Hanna Sabat* from the center of theoretical physics and astrophysics for his help in editing the manuscripts, many thanks to our referees for their valuable comments and notes. We thank all authors who contributed their articles for this issue.

Ammar Sakaji

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Projective Limits of State Spaces: Quantum Field Theory Without a Vacuum

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Abstract: Instead of formulating the states of a Quantum Field Theory (QFT) as density matrices over a single large Hilbert space, it has been proposed by Kijowski [20] to construct them as consistent families of *partial* density matrices, the latter being defined over small 'building block' Hilbert spaces. In this picture, each small Hilbert space can be physically interpreted as extracting from the full theory specific degrees of freedom. This allows to reduce the quantization of a classical field theory to the quantization of *finite-dimensional* sub-systems, thus sidestepping some of the common *ambiguities* (specifically, the issues revolving around the choice of a 'vacuum state'), while obtaining robust and well-controlled quantum states spaces.The present letter provides a self-contained introduction to this formalism, detailing its motivations as well as its relations to other approaches to QFT (such as conventional Fock-like Hilbert spaces, path-integral quantization, and the algebraic formulation). At the same time, it can serve as a reading guide to the series of more in-depth articles [27, 28, 29, 30].

Keywords: Quantum Field Theory; Vacuum States; Inequivalent Representations; Geometric Quantization; Projective Limits; Algebras of Observables
PACS (2010): 02.40.Yy; 03.50.-z; 03.70.+k; 04.62.+v; 04.60.Ds

1. Motivation: Quantization Ambiguities in Quantum Field Theory

Many choices have to be made in the quantization of a classical theory. Assuming one is following the canonical quantization path (see section 5. for further discussion of the relevance for path-integral approaches of the issues discussed here), the first step is to choose a complete set of basic variables for the theory. Heuristically, these are the vari-

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ables for which the semi-classical limit will work best, hence their choice should ideally reflect the observables against which the classical theory of interest has been best tested and confirmed.

The next step is to find a representation of these basic variables as operators on a suitable Hilbert space $\mathcal{H}$, namely a mapping $f \mapsto \hat{f}$ such that

$$[\hat{f}, \hat{g}] = i\,\widehat{\{f, g\}} \tag{1}$$

(where $[\,\cdot\,,\,\cdot\,]$ denotes the commutator of operators, while $\{\,\cdot\,,\,\cdot\,\}$ denotes the Poisson brackets of classical observables). At this point, quantum field theory (in a broad sense, namely quantum theories meant to encompass *infinitely* many degrees of freedom) differs crucially from quantum mechanics (dealing with the quantum counterparts of classical systems that have *finitely* many degrees of freedom). The tools from geometric quantization [45] (that we will discuss further in subsection 2.2) provide a clear and detailed understanding of the canonical quantization of finite dimensional systems, including a parametrization of available choices (aka. quantization ambiguities). In some cases, it may even turn out that there is no choice at all, because the Poisson-algebra of interest admits only one suitable representation: this is for example the content of the Stone-von-Neumann theorem [41, 42, 39] in the case of linear systems.

By contrast, the representation theory for infinite dimensional system tends to be very involved. Even in the simplest case of a free scalar field on Minkowski spacetime, it is known that there exist infinitely many *inequivalent* representations, and although it has been possible, in this very special case, to fully classify them [15], this classification is so complex that it gives little insight on how to choose one. As a way out, a pragmatic way of selecting a good representation among these too numerous options is to single out a distinguished quantum state, the *vacuum*: it is indeed possible, via the so-called GNS construction [11, 38] to 'seed' a full representation $\mathcal{H}_\Omega$ from a single state Ω (to specify the latter, even before we are equipped with a Hilbert space, we can give the corresponding expectation values of all products of the basic variables, aka. the n-point functions, see [17, part III, def. 2.2.8]). This approach has established itself as the standard way to think about quantum field theory, at least in the context of Minkowski spacetime, where the vacuum may be selected by requiring it to be invariant under all spacetime symmetries (ie. under the Poincaré group).

However, one should keep in mind that the initial choice of vacuum is deeply imprinted in the thus obtained representation. The only quantum states that can be written as (pure or statistical) states on $\mathcal{H}_\Omega$ are those that barely differ from the vacuum: at most discrete quantum excitations on top of the state Ω are allowed. The set of all states living on the representation $\mathcal{H}_\Omega$ is referred to as the vacuum *sector*, in acknowledgment of the fact that there are many more quantum states beyond it (falling out of it because they lie too far away from the chosen vacuum), among whose some may actually be interesting for specific purposes [17, part V]. An implication of the relative smallness of the vacuum sector is that the vacuum state need to be closely tailored to the *dynamics*: otherwise, the time evolution would immediately kick the states out of $\mathcal{H}_\Omega$ (a precise statement of this heuristic expectation is given, for Poincaré-invariant QFTs, by the Haag no-go theorem, [16]).

A radical alternative, prompted by the lack of a natural vacuum in the case of quantum field theory on *curved* spacetime, is to use as state space the *whole* set of possible quantum states over the chosen basic observables (each such state being specified, as explained above, by the expectations values it prescribes for all products of observables). This approach can be followed in the context of Algebraic Quantum Field Theory (AQFT, [17, 18]): by shifting the focus from a particle picture to the *local* and *causal* structure of the quantum theory, AQFT provides tools to discuss the properties of quantum fields in the absence of an underlying Hilbert space. The aim of the present letter is to argue that a projective definition of quantum field theory, as was introduced by Jerzy Kijowski [20] and further developed by Andrzej Okołów [32, 34, 33], can provide a middle way between the conventional vacuum-based approach and the full algebraic one, retaining a *constructive* description of the quantum state space (subsections 3.2 and 3.3) while keeping enough *flexibility* to accommodate a wide class of quantum states (subsection 3.1) and to decouple the subsequent implementation of the dynamics from the initial building of the state space (section 4.).

The work summarized in the following sections (and developed in details in [27, 28, 29, 30]) was notably motivated by the specific difficulties encountered when one tries to formulate *background independent* quantum field theories, rather than theories on a (possibly curved) background spacetime (eg. to quantize general relativity itself in a non-perturbative way [2, 40]). It turns out that for background independent gauge theories (at least those with *compact* gauge group), there does exist a *preferred* vacuum state, the Ashtekar-Lewandowski vacuum [3, 4], which is uniquely selected precisely by the requirement of background independence [31, 10]. Unfortunately, this vacuum has some unwanted properties. One of them is that it is an *eigenstate* of the variable conjugate to the gauge field, rather than a *coherent state* like the usual Fock vacuum. Since states in the vacuum sector cannot differ too much from the vacuum, this makes it difficult to find semi-classical states among them [12, 22]. Another problem is that the GNS representation built on this vacuum lives on a *non-separable* Hilbert space. This particular issue may or may not go away once we identify quantum states that only differ by a change of coordinates (depending on how precisely this identification is carried out, see [40, 8]) but in any cases it can lead to technical difficulties [5]. Paradoxically, non-separable Hilbert spaces seem too *small*, because their orthonormal basis need uncountably many basis vectors, making it tempting to consider uncountable linear combinations while only countable ones are allowed: in other words, it is in this case even more likely that physically interesting states will lie out of the vacuum sector. In an effort to overcome these difficulties, the projective quantization techniques that we will review in the next section have been applied to this kind of theories, resulting in a quantum state space that may have applications to the study of the semi-classical and cosmological sectors of quantum gravity [26].

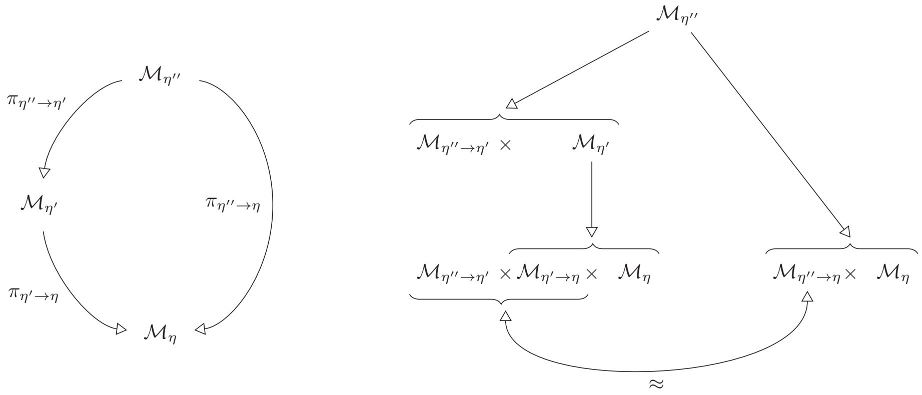

Fig. 1 Three-spaces consistency for projective systems (left side), reformulated in terms of factorizations (right side)

2. Systematic Quantization of Infinite-dimensional Systems

2.1 Building an Infinite-dimensional Theory from a Collection of Partial Descriptions

The key observation underlying Kijowski's projective formalism [20, 33] is that a given experiment can only measure finitely many observables. Thus, we never need to consider at once the full, *infinite*-dimensional phase space $\mathcal{M}_\infty$ of a field theory: it is sufficient to work in a small, partial phase space $\mathcal{M}_\eta$ that extracts from $\mathcal{M}_\infty$ just the degrees of freedom (dof.[2]) relevant for the experiment at hand (throughout the present letter, the symbol η will be used to denote a selection of finitely many dof. out of the full theory, and we will call η a *label*).

In order to use such a collection of finite-dimensional partial phase spaces $\left(\mathcal{M}_\eta\right)_\eta$ to completely specify a field theory, we need to ensure that the different partial theories are consistent with each other [27, subsection 2.1]:

1. first, we need a way to express the relations between the dof. in different labels. We will write $\eta \preccurlyeq \eta'$ if all dof. contained in η are also contained in η' (we will also say that η is *coarser* as η', or that η' is *finer* as η). This means that any observable f_η on $\mathcal{M}_\eta$ corresponds to an observable $f_{\eta'}$ on $\mathcal{M}_{\eta'}$, and, by duality[3], that there exists a projection $\pi_{\eta'\to\eta}$ from $\mathcal{M}_{\eta'}$ to $\mathcal{M}_\eta$ such that

$$f_{\eta'} = f_\eta \circ \pi_{\eta'\to\eta}$$

2. the predictions for a given experiment, as calculated in a partial theory η, should be independent of the choice of η (provided η is fine enough to hold all relevant dof.). Thus, in particular, the Poisson brackets between two observables f_η and g_η on $\mathcal{M}_\eta$

[2] By a dof. we mean a *pair* of conjugate variables.

[3] To see that $\pi_{\eta'\to\eta}$ is uniquely specified once we know the mapping $f_\eta \mapsto f_{\eta'}$ between observables, one can consider a complete set of observables (aka. coordinates) on $\mathcal{M}_\eta$.