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In memory of my father Franco Jossa



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Paolo Jossa

# Prediction Techniques in Classical Physics

The role of dimensional analysis





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## Introduction

### The reasons of this book

This book was inspired by some thoughts that I developed in a previous work<sup>1</sup>. These thoughts, however, have been evolved over time, leading to a goal quite different from the one of the above-mentioned book, which has required a much more broad and differentiated effort.

Here I do not look only at the occasions to give space, in the specificity of a discipline, to the simplifying role of ordinary language. Instead, I am interested to identify a whole series of *simplification techniques for the prediction of scientific results*, as I will tell you soon. And, really, these techniques have become the main reason of this book.

Regarding ordinary language, I now mainly use it with the precise function of guide for prediction, as more specifically you will see in the second chapter. Finally, I extend here the role of dimensional analysis, emphasizing its correlation with the above-mentioned techniques and with ordinary language.

After this, I must be sure that the reference to ordinary language does not cause misunderstandings. This reference, in fact, seems to imply certain themes already studied in-depth, on which I do not want to return. Therefore, I have to defend myself against this risk, and I will do this by starting to state the issues that *not* will be object of my reflection.

I do not want to discuss whether it is possible, or not, the translation of the formalized language of Physics in ordinary language. It would be a return to an all-too-debated issue. Just remember, with G. Battimelli<sup>2</sup>, as a famous figure in the history of Physics, *Michael Far-*

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1. Jossa, P. “*Linguaggio ordinario e ragionamento puro nella Meccanica strutturale*” (*Ordinary language and pure reasoning in Structural Mechanics*), Aracne Publishing, June 2011.

2. Battimelli, G. “*Pillole pedagogiche ovvero I tormenti dell'insegnante di Fisica*”, (*pedagogical pills or the torments of the teacher of Physics*), [www.mercati esplorativi.com](http://www.mercati.esplorativi.com) 2013.

*aday*, wrote, already in 1857, to an equally distinguished colleague, his compatriot *James Clerk Maxwell*: "There is one thing that I would like to ask you. When a mathematician is committed to investigate actions and physical results and comes to his own conclusions, it is not possible to formulate these conclusions in common language, with the same thoroughness, clarity and definiteness that in mathematical formulas? ... ". Besides, I am not interested to investigate, at a general level, on the existing relation between ordinary and mathematical language. This is another widely discussed issue. Moreover, it seems to me that there is now widespread agreement in judging that a complementary relation between these two languages exists, with strengths and weaknesses, as appropriate, for each of them.

I prefer, vice versa, to support my reflection on a motto that has guided, for many years, my teaching of Structural Engineering in the Faculties of Architecture of Naples and Reggio Calabria. The motto was: "*What can be done should not be taught.*" You should read in it the intention of teaching a specialized discipline maximizing the students' activity, so obeying to a *constructivist line*, conglomerate of different positions in the students training.

This choice I practiced in classroom, accompanying it with pauses for reflection. With it I wanted to solicit the initiative, encouraging students to use the means at their disposal. However, that choice was not only the *qualitative reasoning* about the use of acquired knowledge, but it was also solicitation to perform autonomous constructions. A goal maybe difficult, but certainly not impossible to achieve, especially if we are able to produce *self-confidence*, with recognition of the naturalness and the simplicity of many issues.

The reader will notice that the last sentence broadens the discourse, because the words "natural" and "simplicity" connote the mode of action of *common sense*.

However, here, again, I have to say that I am not interested in returning to the question, also this discussed at length, of the potentiality of common sense. I merely point out that common sense is the dowry that more explicit the initiative, because of its natural tendency to look in all directions. In this regard, let me add, and I will reiterate it later, that I refer here to the common sense of a person with a certain level not only of culture, but also of specialist knowledge.

The last clarification comes from the fact that what I will say has *a character in some way preliminary and complementary to a systemat-*

*ic study of some aspects of Classical Physics*, i.e., to a study that normally obeys to the forms proposed by manuals. It is then obvious that I can hardly arouse the interest of a reader away from the culture of science, although, as they say, hope is the last to die.

So far, I have connected the memory of students in classroom with the figure of the reader, with reference to possible opportunities of activity and initiative. I must now be more precise about what I want to do. To this end, it is perhaps appropriate to bring attention, once again, to the few words that I have remembered of the Faraday's speech.

Faraday focuses on the transmission of the conclusions of a research work, a problem that usually, it must be assumed, causes difficulties and time-consuming for the achievement of a result. He therefore waives to describe the discovery path. This is an obvious fact, since discovery is essentially individual, and follows not predictable paths that are privilege of a few. He merely refers to the question, as I said, whether it is possible the translation of the language of mathematics in the language of all, implying the ability of the reader to understand the translation.

I, vice versa, have no knowledge, as obvious, of what have been the real discovery paths. Therefore, *I prefer to place myself in some way in parallel with them*. A location that it is not ambition of reconstruction, because reconstruction is the task of historians and because the historical reconstruction, although sometimes of great interest, is too often forced to lose effectiveness, due to necessary descriptions of returns and errors. Nor yet I have ambition to comparison. It would be an underestimation of the importance and of the originality of certain results, with the advantage of working *ex post*. This is what, for example, I reiterate when I predict some aspects of Maxwell's equations.

What I would like to do it is to verify if it is possible to find new simple ways to conquer a result. With the belief that in such a way we exclude the risk of impoverishing the study of Science with the reduction of the discourse in a rigid conclusive frame, *mere acknowledgment of previous successes*. And here, I must say again to Battimelli<sup>3</sup> that the boredom that some scientific textbooks convey is not always due to a certain poverty of formalized languages, but it can sometimes

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3. Battimelli, G. "*Linguaggi scientifici e linguaggi dei manuali: il caso della Fisica*", (*Scientific languages and languages of manuals: the case of Physics*), [www.phys.uniroma1.it](http://www.phys.uniroma1.it) 2013.

depend on a too systematic organization of the various arguments, which results only in learning, with absence of any autonomous construction.

In short, I think that *we must take advantage of working ex post, by preceding the systematic acquisition of knowledge by one stage of prediction: i.e. from the logical structures, sometimes hidden by developments, and from the facts of common experience that can contribute to explanation.* All this, as I will say, looking continuously at possible simplified paths. A prediction which does not want to describe the mode of formation of a specific scientific result, but tries to grasp, in whole or in part, a possible path of recognition. It is clearly implied in this operation, the ambition to equip the reader so that he could grasp the conceptual simplicity and the naturalness that are usually at the base of a formulation, of a problem, of a theorem and of other aspects of Classical Physics.

E. E. Toulouse<sup>4</sup>, a psychologist of the Psychology Laboratory of the School of Higher Studies in Paris, recall in a book of the year 1910 entitled *Henri Poincaré*, that Poincaré did not care much about rigor, disliking logic. He judged that logic is not a way to invent, but a way for structuring ideas. Moreover, he believed that logic can limit ideas. Toulouse also reminds that the way of Poincaré to deal with a problem was to try solving it in a manner as complete as possible in his mind, and then to translate it into a well organized paper.

Now, it is obvious that we cannot take for example a genius as Poincaré. However, it is a fact that logic is not intuition and is not discovery; and that the immersion, without deviations, in mathematical developments may produce automatisms, limiting the effectiveness of constructive thought. Furthermore, it is evident that the search for unconventional approaches to problems can stimulate the mental activity. Finally, we must not forget that, if it is only highlighted the instrumental nature of a problem, then there is risk to lose the concept and the rationality supporting it.

Dewey says: "The concept has also the feature of allowing the anticipated solution of problems." <sup>5</sup> Therefore, it is also in searching the

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4. Toulouse, E. "*Enquete medico psychologique sur la superiorité intellectuelle: Henry Poincaré*", (*Inquiry Medico psychologique on the intellectual superiority: Henry Poincaré*), E. Flammarion, Paris, 1910.

5. DEWEY, J. "*Logic: the Theory of Inquiry*", Holt, Rinehart and Winston, 1938.