MAGISTRALIA

FILOSOFI DEL TERZO MILLENNIO – QUAESTIONES

1

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Lo scopo della collana, nella sua parte delle Quaestiones, è quello di voler creare un polo di riferimento per tutti i ricercatori e docenti che vogliano approfondire il complesso rapporto tra filosofia e scienza, il dialogo interculturale e interreligioso, nelle loro implicazioni e applicazioni all'orizzonte scientifico e metafisico contemporaneo. A tale scopo vengono raccolte e pubblicate opere di elevato profilo tecnico e scientifico indirizzate agli specialisti del settore che possono trovare collettanee e atti dei convegni attinenti al settore scientifico e disciplinare cui questa collana è indirizzata. L'alto profilo che si va a delineare vuole presentarsi e proporsi come una nuova sistematizzazione dei principali temi e snodi teoretici del dibattito filosofico contemporaneo, forte della sapienza del passato ma con un puntuale e prudente sguardo al futuro, nel tentativo di attualizzare al meglio le potenzialità di questo incredibile settore del Sapere, a cui, soprattutto oggi, non si può non guardare.

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(Un–)Certainty and (In–)Exactness

Proceedings of the 1st CLE Colloquium
for Philosophy and Formal Sciences

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Foreword

The publication of this book follows the *1st CLE Colloquium for Philosophy and History of Formal Sciences* (*1stCLE4Science*), which took place in Campinas, Brazil, from the 21^{st} to the 23^{rd} of March 2013. This Colloquium was organized under the auspices of the "Centre for Logic, Epistemology and the History of Science (CLE)" at the University of Campinas (UNICAMP), with the financial support from the "São Paulo Research Foundation (FAPESP)" and the "Brazilian Federal Agency for Support and Evaluation of Higher Education (CAPES)", which made possible the presence of the visiting professors.

The main goal of "CLE4Science-Colloquia" is to promote regular meetings enabling collaboration among researchers interested in Philosophy and History of Formal Sciences, i.e., branches of knowledge concerned with formal systems, such as Logic, Mathematics, Systems Theory, Information Theory, Cognitive Sciences, Physics, Probability, etc. The theme of the *1st CLE Colloquium for Philosophy and History of Formal Sciences* was "(Un)certainty and (In)exactness".

Our invited speakers presented contributions describing original and unpublished results related with the theme the of the Colloquium's areas. As an opportunity to strengthen the collaboration of research among groups of the "Centre for Logic, Epistemology and the History of Science (CLE)" at the University of Campinas (UNICAMP), and the "International Research Area on Foundations of the Sciences (IRAFS)" at the Pontifical Lateran University (PUL), this book was published in Brazil by "Coleção CLE" and in Europe by "ARACNE Editrice S.r.l.", in order to offer to a wider public the *Proceedings* of the *1stCLE4Science*, containing papers originated at the Colloquium, but often developed by Authors during these last years.

The editors would like to express their sincere gratitude to all participants and authors who accepted the invitation to contribute to this volume and had great patience during the publication process.

Finally, we are proud that this volume inaugurates the Series *Quaestiones* in the Collection *Magistralia* of the prestigious Publishing House “Aracne Editrice S.r.l.”. It is our intent that other volumes will follow the present one, as a testimony of the fruitfulness of the common research effort that CLE and IRAFS want to develop together on the most advanced topics in logic and philosophy.

The Editors
Fabio Bertato
Gianfranco Basti

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I

Russell's criticisms of Euclid, and the nature of proof

IRVING H. ANELLIS*

For nearly 2000 years, until well into the 19th century, Euclid's *Elements* were conceived as providing the most perfect example of rigorous deductive reasoning.

In the late nineteenth and early twentieth century, with increasing attention focused on non-Euclidean geometries Euclid's *Elements*, almost sacrosanct since antiquity, received renewed and critical attention. For example, in a review in 1891 of the English translation (1891; 1892) by George Bruce Halsted (1853–1922) of "The Non-Euclidean Geometry" of Nikolai Ivanovich Lobachevskii (1792–1856) (Lobachevskii (1887)), Charles Sanders Peirce (1839–1914) described Euclid's *Elements* as "unmathematical", writing in his review that: "The truth is that elementary geometry, instead of being the reflection of human reasoning, is riddled with fallacies, and is thoroughly unmathematical in its development. It has in the same measure confused all mathematics, by leaving unnoticed most of the really fundamental propositions, while raising to an undue rank certain others almost arbitrarily selected…." (Peirce (1892)). And in the paper "The Teaching of Euclid" (Russell (1902a))[1], Bertrand Russell (1871–1970) similarly took Euclid seriously to task for the lack of the "logical excellence" which Euclid was

* Indiana University – Purdue University at Indianapolis . The esteemed scholar and historian of logic Dr. Irving H. Anellis passed away on the 15th of July, 2013. We are proud to publish as posthumous this paper that is one of the last works of this outstanding scholar.

[1] For discussions of the historical background to Russell's attitudes towards Euclid as taught

reputed to have presented in his *Elements*, and which had traditionally been ascribed to him since antiquity.

I survey and analyze the criticisms of Euclid, with particular attention to Russell's critique, against the backdrop of metamathematical concerns for the founding of mathematics upon an axiomatic basis and efforts to understand the nature of *proof*. What is crucial during this period is the recognition of a distinction between an axiomatic system (such as Euclid's and Peano's), and a formal deductive system (such as Frege's, Hilbert's, and Russell's).

In his Russell (1902a) "The Teaching Euclid" Russell argued that the nearly 2000-year tradition of proclaiming Euclid's *Elements* as the paradigm and exemplar of careful, rigorous, logical reasoning is incorrect. To the contrary, Russell argued that some of Euclid's proofs were erroneous, and that some alleged demonstrations were not really even proofs at all, properly so-called. This is quite apart even from the question of the correctness and independence of Euclid's controversial Fifth Postulate, the Parallel Postulate, and the possibility or impossibility of non-Euclidean geometries. Rather, Russell's criticism of Euclid hinged upon the nature of proof.

I begin by appealing to the distinction, raised by Jean van Heijenoort in his criticism of Giuseppe Peano's (1858–1932) *Arithmetices Principes*[2] (Peano (1889a)), between an *axiomatic system* and a *formal deductive system*. To render that distinction in its most simplistic terms, a formal deductive system is an axiomatic system which has explicit, and explicitly stated, inference rules. Expressing the conclusion that Russell drew, albeit not explicitly in reference to Peano's *Arithmetices Principia*, but in the terminology adopted from van Heijenoort's critique of Peano, Russell would assert that what Euclid presents in the *Elements* is an axiomatic system, but not a formal deductive system.

The problem is illustrated by Peano's (1889a, p. 2) formula $11\colon 2 \in N$ and the "*Demonstratio*" that follows. "What is presented as a proof,"

in Britain, and the changing attitudes towards non-Euclidean geometries in England in the second half of the nineteenth century chronicled and described, see, e.g. Richards (1988a) and Moktefi (2007; 2011). Richards (1988b) examines in depth the epistemological context and significance of Russell's (1897) *Essay*. Richards (1988a, pp. 201–229) is devoted to "Russell and the Cambridge Tradition".

[2] See van Heijenoort (1967, p. 84) for van Heijenoort's criticism of Peano's lack of an inference rule in his "Introduction" to the translation. Borga and Palladino (1992, p. 22), argue, however, that Peano although treating his equivalent of *modus ponens* as an axiom, nevertheless can be understood as serving as an inference rule for the *Arithmetices principia*.

van Heijenoort explained, "is actually a list of formulas that are such that, from the point of view of the working mathematician, each one is very close to the next. But, however close two successive formulas may be, the logician cannot pass from one to the next because of the absence of rules of inference" (van Heijenoort (1967, p. 84)). Then: "The proof does not get off the ground." Peano's so-called "proof" of formula 11, taken as a theorem, is, van Heijenoort thought, typical of the remainder of Peano's ostensible "proofs". The passage from lines 1 and 2 to line 3 in Peano's *demonstratio* that $2 \in N$, "cannot," van Heijenoort claimed, "be carried out by a *formal* procedure;" rather, "it requires some intuitive logical argument, which the reader has to supply." And it illustrates the difference between a mere *axiomatization* and a *formal deductive system*.

The culprit is the missing rule of detachment; Peano's $a \supset b$ is given by Peano to mean "ab *a* deducitur *b*". But this is left vague. It is not, strictly speaking, van Heijenoort claimed, an inference rule properly so-called. Interestingly, Arthur Richard Schweitzer (1878–1957), a student at the University of Chicago and one of the foremost of the American Postulate Theorist Eliakim Hastings Moore (1862–1932), complained Schweitzer (1914, p. 69), referring to §1 of Russell's *Principles of Mathematics* (Russell (1903, p. 1)), that Russell failed as well to distinguish deduction from inference, although Schweitzer failed to give an explanation, relying upon Russell's asserting, that propositions of the form "*p* implies *q*", in addition to being definable by implication, are also definable in terms of "the relation of a term to a class of which it is a member, the notion of *such that*, the notion of relation, and such further notions as may be involved in the general notion of propositions" of that form.

This did not deter Russell (1903, p. 187) from protesting, however, against Peirce's use of his illation sign (—<) and Schröder's Subsumption (€) for set membership, implication, and class inclusion when arguing the superiority of Peano's notation over Peirce's, with its distinction between set elementhood ($\in$) on the one hand and class inclusion or implication ($\supset$) on the other.

The failure to articulate with care the distinction between deduction, which in the *Principles* Russell calls "*formal implication*", from material implication, and notes that making this distinction is a "difficult business" and depends upon the role of variables, he illustrates by appeal to geometry (Russell (1903, §15, p. 14)):

> The fifth postulate of Euclid follows from the fourth: if the fourth is true, so is the fifth, while if the fifth is false, so is the fourth. This is a case of material implication, for both propositions are absolute constants, not dependent for their meaning, not depending upon the assigning of a value to a variable. But each of them *states* a formal implication. The fourth states that if *x* and *y* are triangles fulfilling certain conditions, then *x* and *y* are triangles fulfilling certain other conditions, and that this implication holds for all values of *x* and *y*; and the fifth states that if *x* is an isoceles triangle, *x* has angles at the base equal[3].

He then immediately adds—uninformatively—that (Russell (1903, §15, p. 14)):

> The formal implication involved in each of these two propositions is quite a different thing from the material implication holding between the propositions as wholes; both notions are required in the propositional calculus, but it is the study of material implication which specially distinguishes this subject, for formal implication occurs throughout the whole of mathematics.

In the *Principles* (Russell (1903, §15, p. 14)), then, the distinction between formal and material implication still remains quite Peanesque, where formal implication is characterized as holding between propositional functions when the antecedent implies the consequent for all values of the variable. The difference between material and formal implication is characterized by Russell in the *Principles* as the difference between being denoted by *implies* in the case of material implication and by *if…then* in the case of formal implication. In the *Principles* (Russell (1903, p. 16)), the closest Russell came to providing a rule of detachment was by listing implication as an axiom. Thus, in the *Principles*, whatever Marco Borga and Dario Palladino (1992, p. 22) and van Heijenoort (1967, p. 84) have said about the absence of inference rules in Peano's system and about logical laws or axioms "playing the role" of inference rules, holds with equal force in discussing the *Principles*.

[3] The standard English translation, in the classic edition by Thomas Little Heath (1861–1940) for Euclid's fourth and fifth postulates are, respectively Euclid (1926, vol. 1, p. 154): "That all right triangles are equal to one another" and Euclid (1926, vol. 1, p. 155): "That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles."

We could conclude from this treatment by Russell that: (1) material implication belongs to logic, formal implication to all of mathematics; and either that (2) while material implication holds between propositions, formal implication holds between the undefined terms, or variables, of propositions, or (3) that material implication pertains to propositions, formal implication to propositional functions, the latter identifiable by possessing undefined variables. When finally, we pass to Chapter III of the *Principles* wherein Russell (1903, §37–45, pp. 33–41) unpacks what he wrote about the distinction between formal and material implication, we find that, in order to avoid appeal to psychology to distinguish formal from material implication, and thus truth from validity, Russell merely expostulates upon the role of variables in propositional functions and leads us to infer that (2) and (3) are essentially equivalent, and that they tell us all that can be said about how to distinguish material from formal implication. If, therefore, Peano cannot tell the difference between an axiomatic system and a formal deductive system, and takes his "*demonstratio*" of $2 \in N$ as a proof, without the explicit application of any inference rule, then neither, we are led to conclude, can Russell. Either that, or Russell, thanks to his requirement that there can be nothing outside of his logical system, can ultimately tell us nothing, within his logical system, about his logical system.

In writing of Euclid in "The Teaching of Euclid" (Russell (1902a)), Russell pointedly insisted that Euclid's so-called logical proofs depended upon one's mathematical intuition, rather than rigorous formal deduction, and that the intuitive inferences drawn in Euclid relied almost entirely, if indeed not wholly, upon the construction of the diagrams.

Several important, and I think related questions, principally historical but also philosophical, may be raised regarding my reading of Russell's 1902 essay: of (1) whether my interpretation of Russell's criticisms of Euclid reflect what Russell may have in fact had in mind; (2) whether the same criticisms that I claim Russell raised of Euclid could not just as well be directed at other mathematicians; (3) where and how to draw a line between a computation, an axiomatic system, a formal deductive system; and (4) whether Euclid's *Elements* and Aristotle's *Analytics* present deductive systems.

It is easiest to dispense with the first point quickly. Russell's principal theme here was that symbolic logic is the exemplary means for rigorously and systematically developing — as well as teaching — geometry. His concern was not that Euclid made mathematical errors, but that the *Elements* did not present theorems in strict and rigorous accordance with logical reasoning. This is not to say that the propositions presented by Euclid were not properly ordered, and indeed one could readily follow the thought processes that led from one to the next; moreover, each one was in most cases close enough to its predecessor and to its successor to allow an intuitive grasp of the transition from one to the next. But the justifications and explanations that attended the stepwise development were not explicitly based upon logical inferences.

Since Russell's "The Teaching Euclid" (Russell (1902a)) is relatively unfamiliar, I hope I might be forgiven for a comparatively extensive quotation to make my point: Against the concept of Euclid's *Elements* as a masterpiece and exemplar of logical reasoning, because Euclid's "logical excellence is transcendent," Russell began in his essay "The Teaching Euclid" (p. 165) by asserting that this claim "vanishes on a close inspection. His definitions to not always define, his axioms are not always indemonstrable, his demonstrations require many axioms of which he is quite unconscious. A valid proof retains its demonstrative force when no figure is drawn, but very many of Euclid's earlier proofs fail before this text." Among the examples of problems are the first proposition, which assumes, without warrant, the intersection of the circles used in the construction; another example is the fourth proposition, which Russell calls "a tissue of nonsense", given that superposition is "a logically worthless device," and a logical contradiction arises when, taking the triangles as spatial rather than material, one engages the idea of moving them, while, if taking them as material, they cannot be supposed to be perfectly rigid and thus, when superposed, they are certain to be slightly deformed from their previous shape.

For Russell, an early motivation to seek certainty in mathematics was the acceptance by Euclid without proof of his axioms, as Russell recalled (Russell (1967, p. 36)). Studying Euclid's geometry in his mid-teens, Russell began to be plagued by doubts about geometry (as well as differential calculus), and this, in 1888, set him in search, he recalled

(Russell (1889a)) of absolute certainty in mathematics, "Some of Euclid's proofs," he wrote (Russell (1959, p, 27)) "especially those that used the method of superposition, appeared to me very shaky. One of my tutors spoke to me of non-Euclidean geometry. Although I knew nothing of it" at that time, "I found the knowledge that there was such a subject very exciting, intellectually delightful, but a source of disquieting geometrical doubt." In *An Essay on the Foundations of Geometry* (Russell (1897, pp. 54-63)), Russell argued that the development of "metageometry", i.e. the axiomatic foundations of (Euclidean and non-Euclidean) geometries, has shown that Kant's argument for the apodeiciticity of Euclidean geometry breaks down. But Russell did not accept either the position that non-Euclidean geometries are necessary (in any Kantian sense).

Instead, Russell (Russell (1897*,* p. 6)) concluded that only those axioms which are common to both Euclidean and non-Euclidean geometries are *à priori*, whereas the axioms specific to Euclidean geometry are "wholly empirical," as are those axioms specific to the various non-Euclidean geometries. This view was reinforced by Russell's (1898) reply to Louis Couturat's (1868–1914) (Couturat (1898)) review of the *Essay,* in which Russell first accepts Couturat's assertion that Russell's argument in the *Essay* for the empirical character of Euclidean geometry is weak but then defends the empiricality of Euclidean geometry with new arguments.

Russell made the point much more clearly in his *Encyclopedia Britannica* article on non-Euclidean geometry (Russell (1902b, p. 673)) that, although Kant's view that geometry is synthetic permits the possibility that there might be non-Euclidean geometries, "Kant maintained (that there) is *à priori* ground for excluding all or some of the non-Euclidean spaces."

Through most of the history of mathematics, Euclid was accounted the ideal exemplar of sound mathematical reasoning. This was the way it was presented at Cambridge University when Russell studied mathematics. Non-Euclidean geometries were not taught at Cambridge while Russell was a student, although elsewhere, David Hilbert (1862–1943) and others like him were not only examining other geometries and attempting to structure them axiomatically in accordance with explicit inference rules (the American postulate theorists, e.g. Edward Vermilye Huntington (1874–1952), with his "A Set of Postulates for

Abstract Geometry, Expressed in Terms of the Simple Relation of Inclusion” (Huntington (1913)), as well as Italian and German mathematicians, such as Mario Pieri (1860–1913), with his “I principii della geometria di posizione, in sistema logico deduttivo” (Pieri (1898)), “Della geometria elementare come sisterma ipotetico-deduttivo” (Pieri (1899)), and “Sur la géométrie envisagée come un système purement logique” (Pieri (1901)), and Moritz Pasch (1843–1930), with his *Vorlesungen über neuere Geometrie* (Pasch (1882)), were or had been working on sets of axioms for various geometries); and it is against this background and within this milieu that Russell, for his graduate fellowship, undertook his philosophical study of metageometry, published the following year as his *Essay on the Foundations of Geometry* (Russell (1897)).

In preparation for his work on the *Essay*, Russell read everything he could find on geometry and its foundations, concentrating on works that focused their attention on the axioms of geometry. Among the work that Russell read between 1893 and 1896, many more than once, were[4]: Lobachevskii’s *Theorie der Parallellinien* (Lobachevskii (1887)); some of the geometrical works by Bernhard Riemann (1826–1866), especially Riemann’s (published: 1867; but first presented in 1854) “Hypothesen welche der Geometrie zu Grunde liegen” (Riemann (1867)); Felix Klein (1849–1925), Klein’s “*Nicht-Euklid*” (meaning, most likely, both Klein’s (1871) “Ueber die sogennante Nicht-Euklidische Geometrie” and both volume I (Klein (1890)) of *Nicht-Euklidische Geometrie* and (Klein (1893)) *Einleitung in die höhere Geometrie*; Johannes Frischauf (1837–1924), namely his (Frischauf (1872)) *Absolute Geometrie* (essentially a liberal translation of János Bolyai’s (1802–1860) (Bolyai (1832)) *Appendix*, “Science Absolute of Space”, together with comments upon it by Farkas (Wolfgang) Bolyai de Bolya (1775–1856)); Wilhelm Karl Joseph Killing (1847–1923), namely, his *Die Nicht-Euklidischen Raumformen* (Killing (1885)); Benno Erdmann’s (1851–1921) *Die Axiome der Geometrie* (Erdmann (1877)); Hermann Ludwig Ferdinand von Helmholtz’s (1821–1894) “*Sämtliche Schriften über Geometrie*” (evidently a reference to several of Helmholtz’s papers on geometry and philosophy of geometry in

[4] See Russell (1983, pp. 347–365), “What Shall I Read?”, his [incomplete] list of reading covering the period 1891–1902. See also Anellis (1995, p. 105) and Anellis (2006, pp. 22–24) for further details.

Helmholtz (1882-95), and to Helmholtz (1878), rather than to a specific collection by that title); Arthur Cayley's (1821–1895) "Sixth Memoir upon Quantics" (Cayley (1859)); Carl Stumpf's (1848–1936) *Ursprung der Raumvorstellung* (Stumpf (1873)); Carl Friedrich Gauss's (1777–1855) *Disquisitiones circa superficias curvas* (the work of 1828 that founded differential geometry, which Russell presumably saw in Gauss (1880), Eugenio Beltrami's (1835–1899) "Saggio di interpretazione della geometria non-euclidea" (Beltrami (1868a)) and (Beltrami (1868b)) "Teoria fondamentale degli spazii di curvatura costante"; Marius Sophus Lie's (1842–1899) *Grundlagen der Geometrie* (Lie (1890)); Georges Lechalas's (1851–1919) *L'Espace et le Temps* (Lechalas (1896)); Hermann Günther Grassmann's (1809–1877) *Ausdehnungslehre von 1844* (Grassman (1878)); the *Principes de la Métagéométrie* of Paul Mansion (1844–1919) (Mansion (1896)); and the *Hypothèses dans la Géométrie* of Louis Joseph-Florentin Bonnel (or Bonel, 1826–1902) (presumably referring to all but the last installment of Bonnel (1895-1898)).

After completing the *Essay*, he continued into early 1901 to read works in non-Euclidean geometry and foundations of geometry, including, among the writings not already read, works by János Bolyai (evidently the translation by George Bruce Halsted (1853–1922) of "Science Absolute of Space" (Bolyai (1896)); David Hilbert (1862–1943), namely the (Hilbert (1899)) *Grundlagen der Geometrie*; adding the second volume of Klein's *Nicht-Euklidische Geometrie*; Halsted's (Lobachevskii (1892)) translation of Lobachevskii's "Geometrical Researches in the Theory of Parallels"; Pasch's (1882), *Vorlesungen über neuere Geometrie*); Peano's (1889b) *Principii di geometria logicamente esposti*); and along with Pieri's (1898) "I principii della geometria di posizione, in sistema logico deduttivo", and an offprint or preprint of Pieri's 1900 Paris Congress talk (1901) "Sur la géométrie envisagée come un système purement logique").

By this time his primary attention was focused on logic and set theory; and we may note that the very titles of the works of Peano and Pieri which Russell read during this period in particular draw an explicit connection between logic and geometry. Commenting upon this reading Russell wrote that "I discovered that, in addition to Euclidean geometry there were various non-Euclidean varieties, and that no one knew which was right" (Russell (1948, p. 143)).

In the remainder of my reply to the first question, I will tentatively establish an at least partial link between it and question (4).

The question of how to consider Euclid's *Elements* — as an axiomatic system or as a formal deductive system, if we define a formal deductive system as an axiomatic system with explicit inference rules, will depend in part on whether one considers Aristotle's syllogistic logic as providing explicit inference rules (e.g., whether the *Barbara* syllogism is understood, taken in its most general form, as itself an inference rule or as a valid argument structure — or at least functions as if it were an inference rule, but which does not provide more than a psychological and metaphysical explanation of how valid reasoning is to proceed; or if the Laws of Identity, of Non-Contradiction, and Excluded Middle are considered as inference rules or metalogical principles).

The second, purely historical consideration in how Euclid is to be understood has been a matter of debate among specialists. There are essentially two schools of thought on the matter, and so far as I am presently aware, no consensus. One school argues that Aristotle specifically wrote his *Analytics* as a justification for the methods of demonstrations which Euclid utilizes; the other that Euclid deliberately proceeded in the demonstrations in his *Elements* in accordance with the syllogistic rules devised by Aristotle in the *Analytics*. (The sub-question is whether Euclid proceeded in his proofs on the categorical or the hypothetical syllogism).

The one thing both schools agree upon is that, in explaining the mechanics of the syllogism, Aristotle frequently employs geometrical examples to illustrate his points. The possibility of Aristotle undertaking his work in constructing logic as a justification for the method of proof employed by Euclid was dependent upon the older chronology, which had Euclid as Aristotle's contemporary, with his dates thought to have been *ca*. 356–*ca*. 300 B.C.E.[5], Aristotle's being 384/3 (?)–323 B.C.E.). Consider Aristotle's example, in Bk. II, Chapt. 17 of the *Prior Analytics* (Aristotle (1928)): "it is not perhaps absurd that the same false result should follow from several hypotheses, e.g., that parallels meet, both on the assumption that the interior angle is

[5] See, e.g. Lee (1935), Einarson (1936), Corcoran (1973), Gómez-Lobo (1977), and Smith (1977-1978) on the relation of Aristotelian syllogistics to Euclid's proof procedures.