ASTRONOMIA E ASTROFISICA
SCIENTIFICA

COLLANA DIRETTA DA ENRICO COSTA ED ENRICO MASSARO

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La più sublime, la più nobile tra le Fisiche scienze ella è senza dubbio l'Astronomia. L'uomo s'innalza per mezzo di essa come al di sopra di se medesimo, e giunge a conoscere la causa dei fenomeni più straordinari.

Giacomo Leopardi

Negli ultimi anni si è assistito ad una grande crescita di libri dedicati alla descrizione dei primi istanti dell'universo e delle sue complicate proprietà fisiche o alla scoperta di un sempre crescente numero di pianeti in rotazione attorno a stelle vicine.

Gli argomenti trattati nelle ricerche astronomiche spaziano in un panorama molto più ampio, spesso poco noto alla maggioranza dei lettori. Molti dei risultati recenti devono essere confermati ed ampliati e ciò richiede un numero sempre più grande di osservazioni e di accurate analisi dei dati così ottenuti. Accade spesso che le tecniche i dettagli di questi lavori non riescono ad essere descritti come meriterebbero nel ristretto spazio di un articolo su rivista.

Questa collana si prefigge di colmare in parte questa lacuna pubblicando testi che forniscano agli specialisti, come a coloro che affrontano queste impegnative ricerche, una documentazione che ne descriva i diversi aspetti.

Ad essi si affiancheranno anche cataloghi e raccolte di dati, un fondamentale *thesaurus* per le ricerche astrofisiche, e testi più semplici di livello introduttivo.

La collana si divide in due sezioni: in questa sono ospitati i volumi con un taglio e un orientamento scientifico.

Luigi Secco

Galaxy dynamics

Formation and virialization of galaxies
within cosmological environment
Volume 1

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Preface

Within the international AstroMundus path the present course has been developed for many years. The aim is to give some guidelines in understanding the formation and virialization of structures on galaxy scale in the cosmological framework. To understand the cosmological location of the topics considered, we have to look at the main features of the Universe throughout its evolution. We immediately realize that a fundamental epoch divides its history into two parts: the Recombination epoch. Before it the Universe was engaged in structuring itself at a microscopic level. The quarks–adrons phase transition at about 10^{-5} sec after the Big–Bang or the primordial nucleosynthesis within the first 3 minutes, may be considered as relevant exemplifications of that trend. After Recombination the aim of the Universe was to structure itself at a macroscopic level, forming macro structures from the scale of *stellar clusters* to that of *galaxy superclusters*. A very interesting interplay exists between the micro and macro structure formations with the complicity of cosmological expansion. Indeed as density and temperature decrease, matter transforms from plasma into neutral matter. A primordial mixture impossible to disintangle moves gradually towards a decoupling of matter and radiation. As a consequence of neutral matter formation, radiation needs much time to interact with matter for this matter and radiation get two different temperatures. The breaking of thermodynamical symmetry occurs, which allows gravity to freely act on the matter without something like *viscosity* due to radiation. Macro structures may form. Then the neutral atom formation allows for that of the macro ones and within theses latests (e.g. inside stars of galaxies), the complexity of micro–structures may increase. The contents of this book begin with the processes of galaxy formation and then they are related to epochs after Recombination. The natural framework used is the cosmological one. So we need to introduce the main ingredients to deal with it. In Chapt. 1 we introduce Robertson–Walker's metric in order to obtain

Cosmic Dynamics, based on Friedmann's equations (Chapt. 2), putting inside the proper metric Einstein's equations (Chapt. 3). We then move on to consider what happens during Recombination (Chapt. 4) and why the thermal capacity of radiation is so great with respect to that of matter (Chapt. 5). In Chapt. 6 we consider in which way a tiny matter density perturbation may increase, according to Jeans' mechanism characterizing the *initial linear regime* of perturbations. Their statistics are taken into account in Chapt. 7. As the *proto–structure* detaches from Hubble flow, the necessary transition from linear to *non–linear regime* has to be developed (Chapt. 8). Here the "top–hat" approximation is considered. Although idealized it enables us to introduce the main concepts. Galaxy evolution does indeed involve higly non–linear density fluctuations. To follow them only numerical simulations are available. Nevertheless, approximate analytic arguments work as a compass to guide the understanding of the former ones. In Chapt. 9 the *virialization phase* is considered after oscillations of relaxation damped by *Landau mechanism*. To understand it we need to introduce Boltzmann's equation for *collisionless particle system* in the $\mu-$ phase space. The whole, still open problem of *Violent Relaxation* is dealt with in Capt. 10. The final purpose is to understand where galaxies land at virialization. In other words to make an interpretation of the *galaxy Fundamental Plane* (Volume 2). The discovery of the *Cosmic Metaplane* which has shown some common features with that of galaxies, allows us to understand the fate for all virialized cosmological structures from *globular clusters* to *galaxy clusters*. The present volume is mostly devoted to theory rather than observations which are taken into wider account in Volume 2. The level of the textbook corresponds to that of a Master Degree in Astronomy and/or Astrophysics. Necessary requirements are: basic knowledge of analytical mechanics and Relativity (at least the Special).

Padova
September, 2016

Chapter I

The Metric

1.1. Transformation by Covariance and Contravariance

1.1.1. *Vectors*

We consider two systems of coordinates (Sokolnikoff, 1964):

$$\begin{cases} X: & x^i = (x^1, x^2, \ldots\ldots x^n) \\ Y: & y^i = (y^1, y^2, \ldots\ldots y^n) \end{cases} \tag{1.1}$$

and the transformation between them:

$$T: \quad x^i = x^i(y^1, y^2, \ldots\ldots y^n) \tag{1.2}$$

We form the set of partial derivatives:

$$\frac{\partial f}{\partial x^1}, \frac{\partial f}{\partial x^2}, \ldots\ldots \frac{\partial f}{\partial x^n} \tag{1.3}$$

of a continuously differentiable function $f(x^1, x^2, \ldots\ldots x^n)$ that is gradient components of a potential function. The same vector in system Y has components:

$$\frac{\partial f}{\partial y^1}, \frac{\partial f}{\partial y^2}, \ldots\ldots \frac{\partial f}{\partial y^n} \tag{1.4}$$

linked to the previous ones by the rule for differentiation of composite functions, namely[1]:

$$\frac{\partial f}{\partial y^i} = \frac{\partial f}{\partial x^\alpha}\frac{\partial x^\alpha}{\partial y^i} \tag{1.5}$$

1. Einstein's convention that repeated indices imply the summation, always holds.

If, in general, the components of a vector in X: $A_1(x), \ldots . A_n(x)$, transform in the system Y as:

$$B_i(y) = \frac{\partial x^\alpha}{\partial y^i} A_\alpha(x) \tag{1.6}$$

we call that a *law of covariant* transformation and use by convenction the subscripts for sets that transform in this manner. Another law of transformation of vectors which is quite different from the previous one refers to the infinitesimal displacement vector: P_1P_2, where

$$P_1 \equiv P_1(x^1, x^2, \ldots ..x^n), P_2 \equiv P_1(x^1 + \mathrm{d}x^1, x^2 + \mathrm{d}x^2, \ldots ..x^n + \mathrm{d}x^n) \tag{1.7}$$

Due to differentation law we have:

$$\mathrm{d}y^i = \frac{\partial y^i}{\partial x^\alpha} \mathrm{d}x^\alpha; \quad (i, \alpha = 1, 2, \ldots .n) \tag{1.8}$$

which yields:

$$B^i(y) = \frac{\partial y^i}{\partial x^\alpha} A^\alpha(x) \tag{1.9}$$

we call that a *law of contravariant* transformation and use by convenction the superscripts for sets that transform in this manner.

Meaning of covariant and contravariant components

To understand the difference, we refer to the Fig 1.1. We make up the basis dual ($\underline{e}^x$, $\underline{e}^y$) to the basis ($\underline{e}_x$, $\underline{e}_y$), in the following way:

$$\underline{e}^y \cdot \underline{e}_x = 0; \qquad \underline{e}^x \cdot \underline{e}_y = 0$$

Noting that: $\underline{e}_x$, $\underline{e}_y$ are unit vectors and imposing:

$$\underline{e}^x \cdot \underline{e}_x = 1 \rightarrow |\underline{e}^x||\underline{e}_x| cos(\pi/2 - \psi) \rightarrow |\underline{e}^x| = \frac{1}{sin\psi}$$

The same holds for: $\underline{e}^y$;

— upper components: a^x, a^y are contravariants:

$$\underline{a} = a^x \underline{e}_x + a^y \underline{e}_y$$

— lower components: a_x, a_y are covariants:

$$\underline{a} = a_x \underline{e}^x + a_y \underline{e}^y \text{ (in the basis dual)}$$

"co is low"!

1.1.2. *Tensors*

A *covariant tensor of rank one* is a set of quantities: $A(1;x)$, $A(2;x)$,..., $A(n;x)$ which transforms from the X–coordinate system into the Y–one, according to:

$$B(i;y) = \frac{\partial x^\alpha}{\partial y^i} A(\alpha, x) \tag{1.10}$$

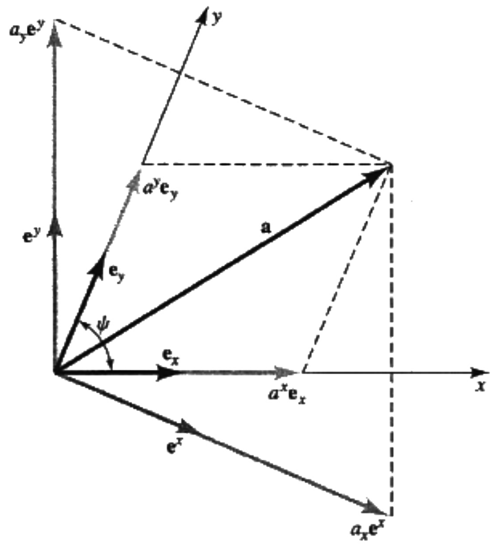

Figure 1.1. Contravariant (a^x, a^y) and covariant (a_x, a_y) vector components in a skew system: $\psi \neq 90^o$. In the case $\psi = 90^o$ they coincide. The basis dual ($\underline{e}^x$, $\underline{e}^y$) is obtained as in the text (Hartle, 2003).

which means (by convention)[2]:

$$B_i = \frac{\partial x^\alpha}{\partial y^i} A_\alpha \quad \text{(covariant law)} \tag{1.11}$$

Moreover a *covariant tensor of rank two* is a set of quantities: $A(i,j;x)$ which transforms from the X–coordinate system into the Y–one, according to:

$$B(i,j;y) = \frac{\partial x^\alpha}{\partial y^i} \frac{\partial x^\beta}{\partial y^j} A(\alpha,\beta;x) \tag{1.12}$$

which by convention will be denoted:

$$B_{ij} = \frac{\partial x^\alpha}{\partial y^i} \frac{\partial x^\beta}{\partial y^j} A_{\alpha\beta} \tag{1.13}$$

On the contrary a set of quantities $A(i;x)$ which transforms from the X–coordinate system X into the Y–one, according to:

$$B(i;y) = \frac{\partial y^i}{\partial x^\alpha} A(\alpha;x) \tag{1.14}$$

which by convention means:

$$B^i = \frac{\partial y^i}{\partial x^\alpha} A^\alpha \quad (\textit{contravariant law}) \tag{1.15}$$

defines a *contravariant tensor of rank one*. Moreover for contravariant tensor of rank two we have:

$$B^{ij} = \frac{\partial y^i}{\partial x^\alpha} \frac{\partial y^j}{\partial x^\beta} A^{\alpha\beta} \tag{1.16}$$

In the case of a tensor which transforms according to:

$$B_i^j(y) = \frac{\partial x^\alpha}{\partial y^i} \frac{\partial y^j}{\partial x^\beta} A_\alpha^\beta(x) \tag{1.17}$$

we will discuss of a *mixed tensor, covariant of rank one and contravariant of rank one*. The extension to higher ranks is manifest.

2. The only exception to this convention is the use of superscripts to identify the variables x^i, y^i, etc. These quantities do not transform according to covariant or contravariant law (see, Sokolnikoff 1964, p. 60).